POINTED PSEUDOTRIANGULATIONS &

A SPHERE OF SPHERICALS

Asilata Bapat

[with Anand Deopurkar,
Anthony M. Licata]

PLAN

$\boxed{\text{Triangulations}}$

PLAN

Pointed pseudo-

Triangulations

PLAN

Pointed pseudo-

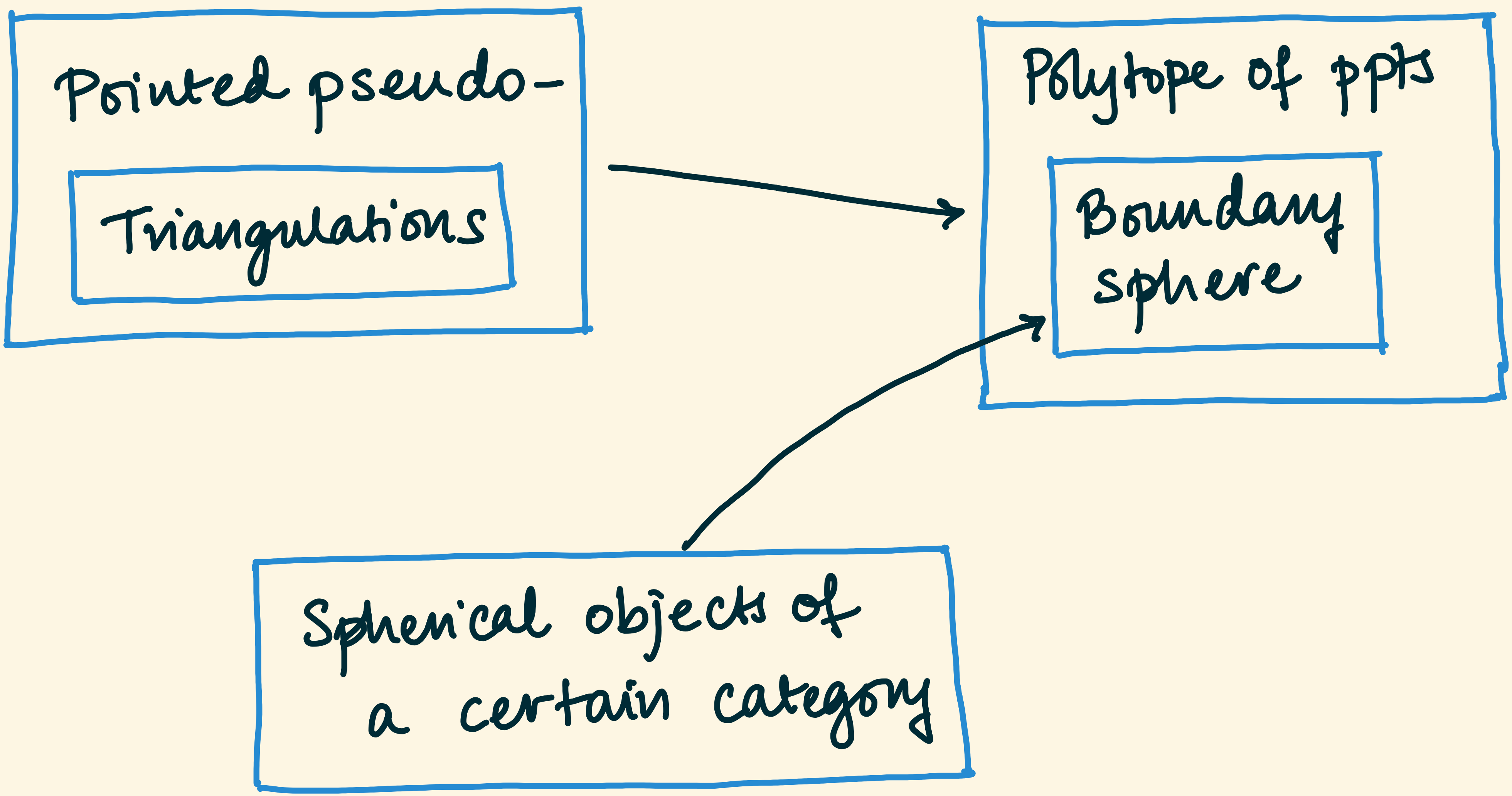
Triangulations

Polytope of ppts

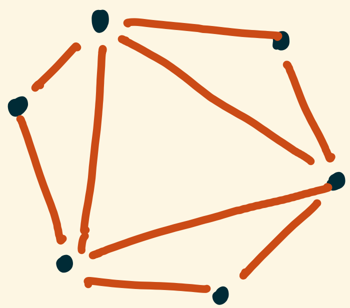
PLAN

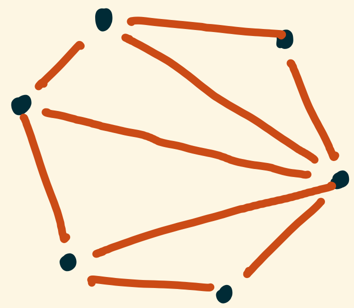
Pointed pseudo-Triangulations $\longrightarrow$ Polytope of ppts: Boundary sphere

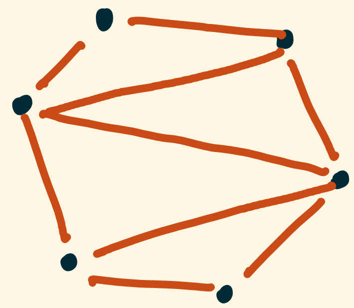
PLAN

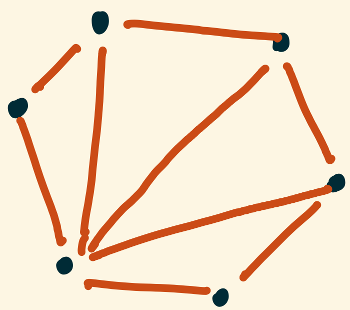

Pointed pseudo-
Triangulations
Polytope of ppts
Boundary sphere
Spherical objects of a certain category

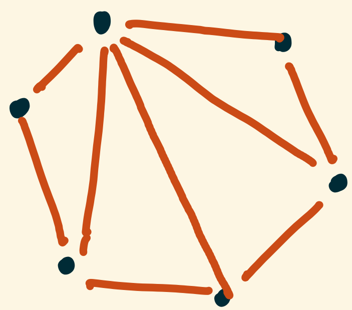
TRIANGULATIONS OF A CONVEX N-GON

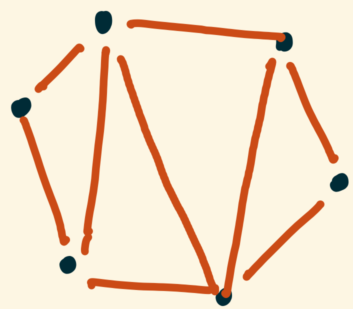












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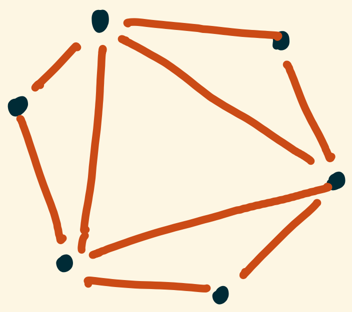
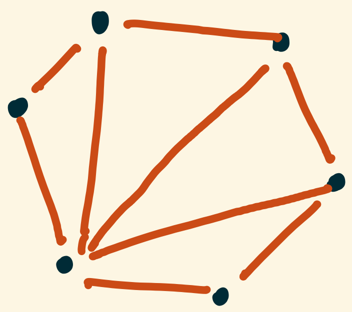


Triangulation

= subdivision into triangles by a maximal number of non-crossing edges.



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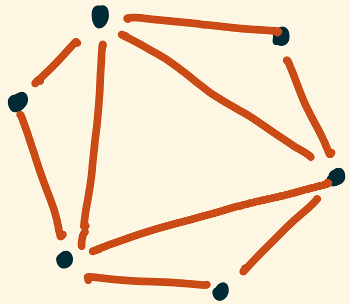
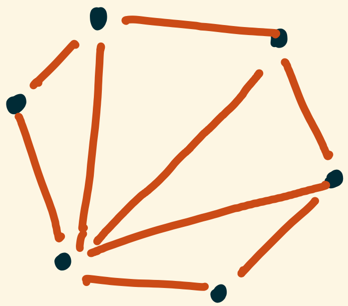
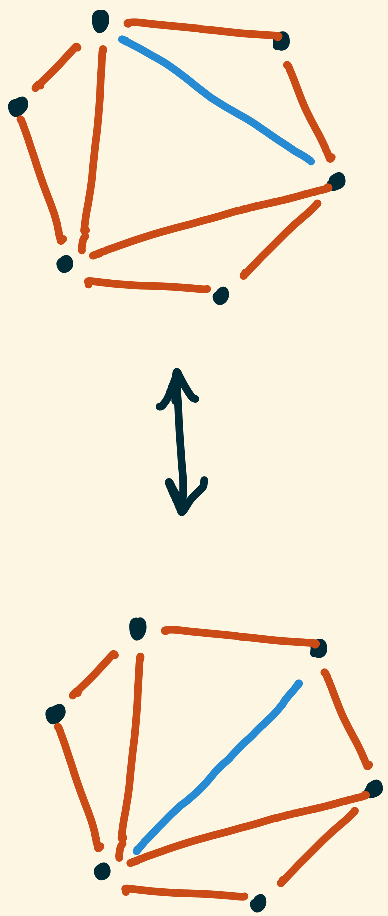


Facts

1) Any triangulation of an n-gon has $(2n-3)$ edges



TRIANGULATIONS OF A CONVEX N-GON

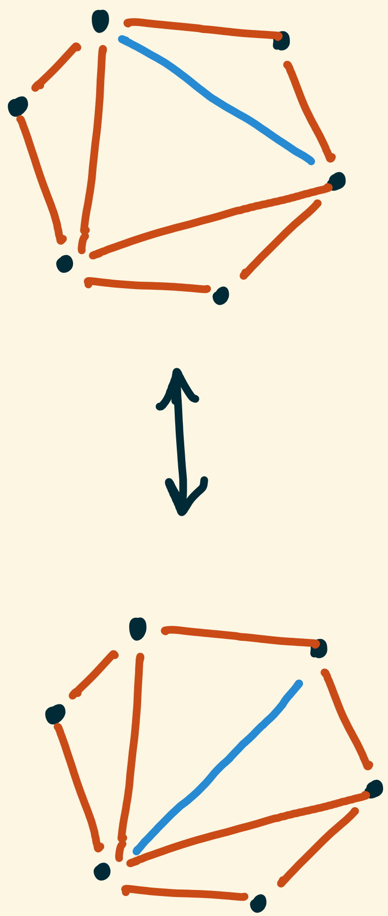


Facts

1) Any triangulation of an n-gon has $(2n-3)$ edges

2) Every internal edge has a unique flip.

TRIANGULATIONS OF A CONVEX N-GON



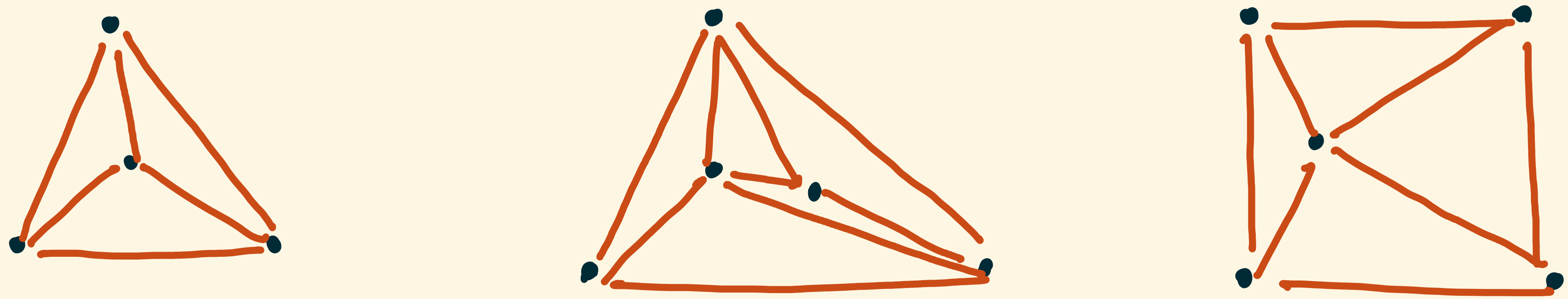
Facts

1) Any triangulation of an n-gon has $(2n-3)$ edges

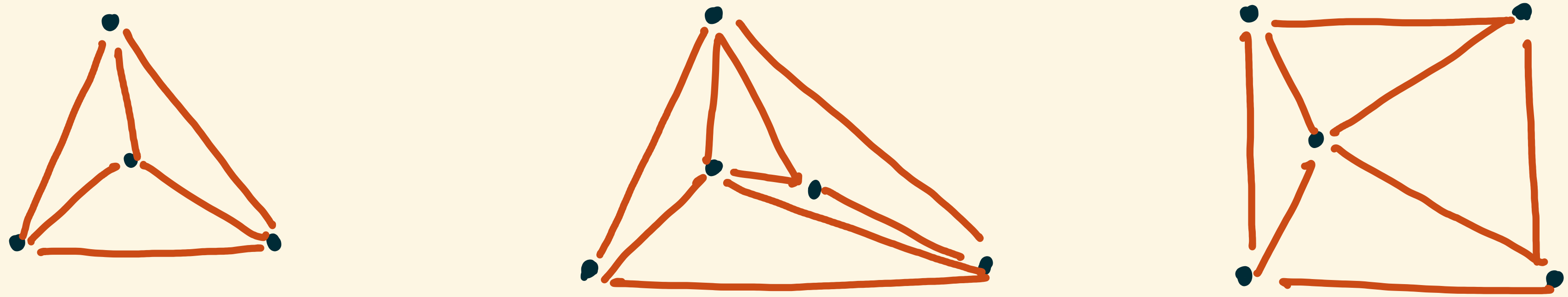
2) Every internal edge has a unique flip.

3) The flip graph is connected.

NON-CONVEX GENERIC CONFIGURATIONS ?

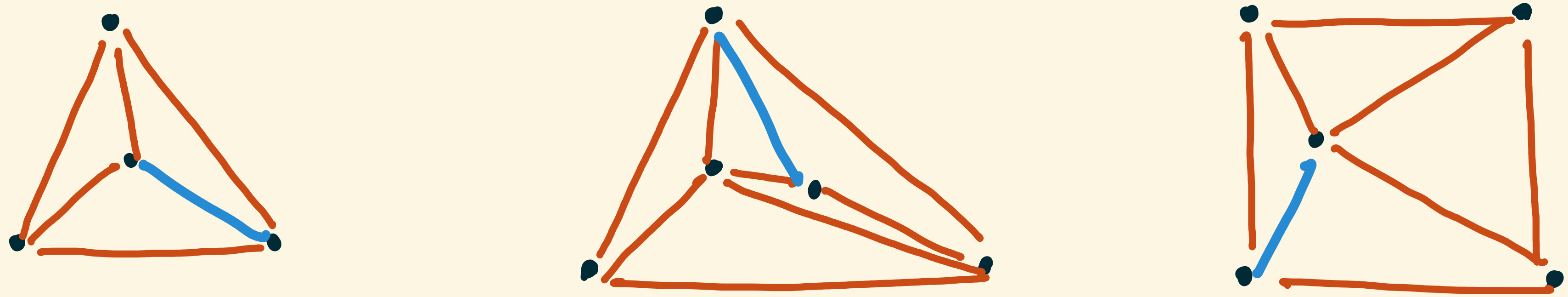


NON-CONVEX GENERIC CONFIGURATIONS?



- Triangulations don't have $2n-3$ edges

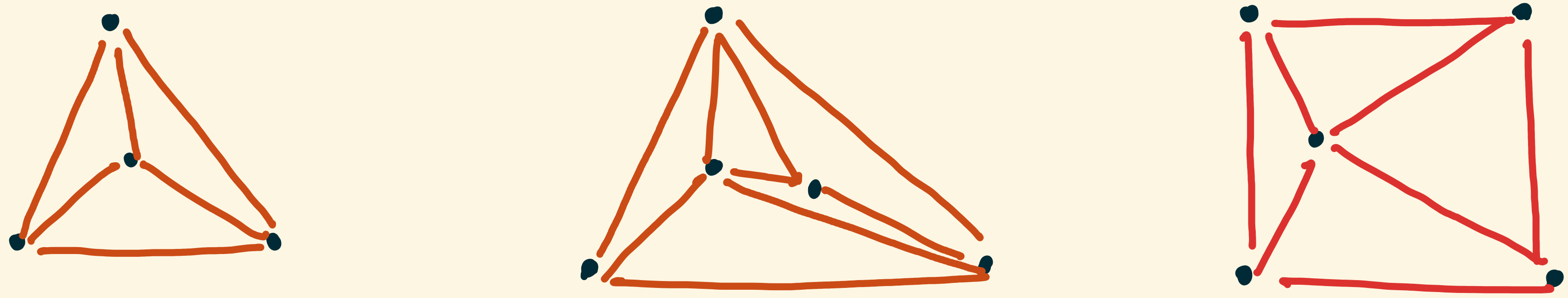
NON-CONVEX GENERIC CONFIGURATIONS?



- Triangulations don't have $2n-3$ edges

- Internal edges not always flippable

NON-CONVEX GENERIC CONFIGURATIONS?



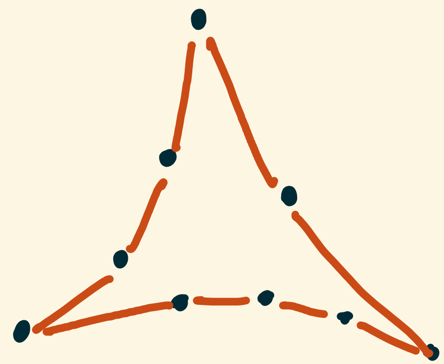
- Triangulations don't have 2n-3 edges

- Internal edges not always flippable

But this can be salvaged!

POINTED PSEUDO-TRIANGULATIONS

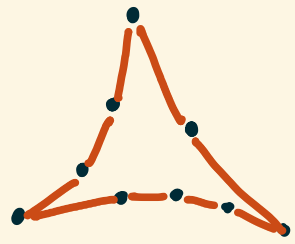
A pseudo-triangle :



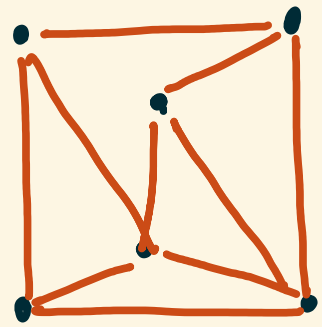
[= Non-crossing polygon whose convex hull is a triangle, which has exactly 3 convex angles.]

POINTED PSEUDO-TRIANGULATIONS

A pseudo-triangle :

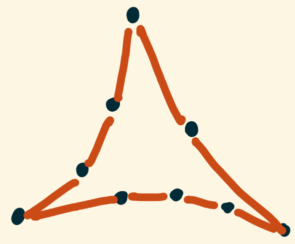


A pseudo-triangulation :

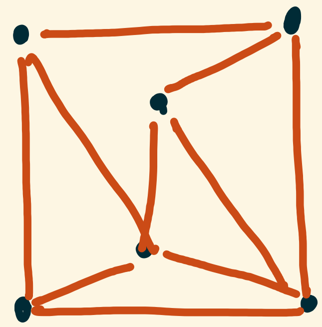


POINTED PSEUDO-TRIANGULATIONS

A pseudo-triangle :



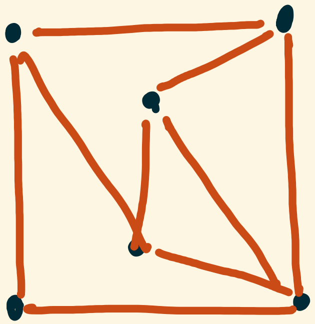
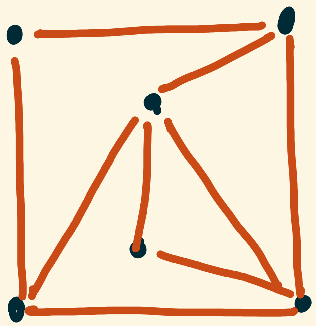
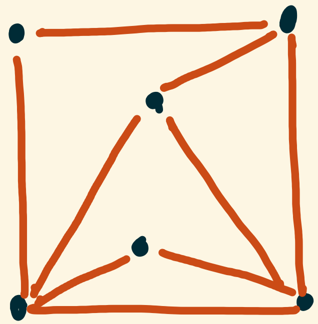
A pseudo-triangulation :



[Any triangulation is also a pseudo-triangulation !]

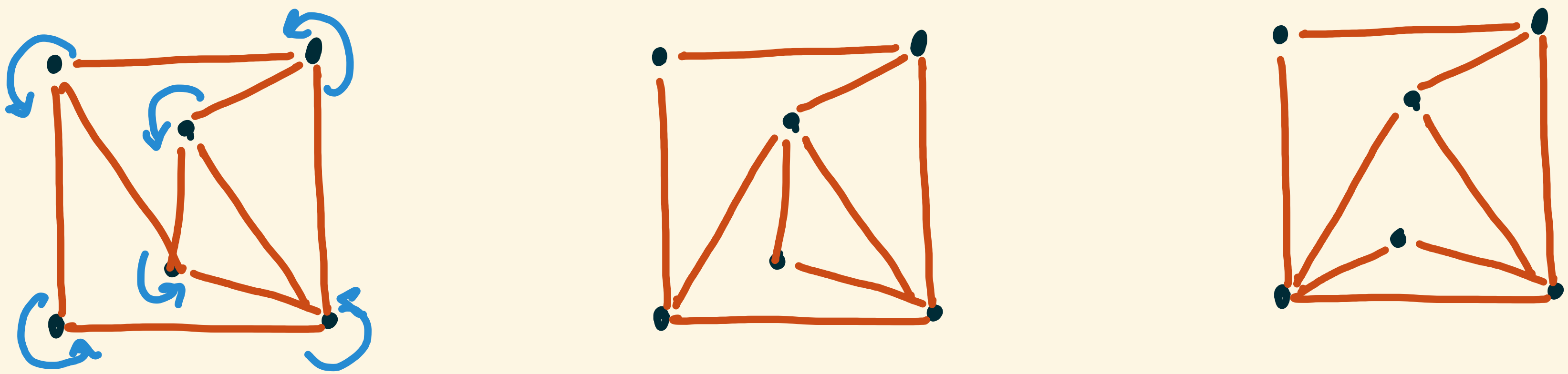
POINTED PSEUDO-TRIANGULATIONS

Pointed pseudotriangulations :

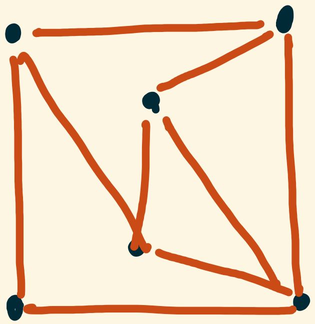
POINTED PSEUDO-TRIANGULATIONS

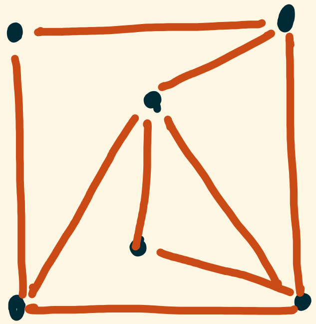
Pointed pseudotriangulations :

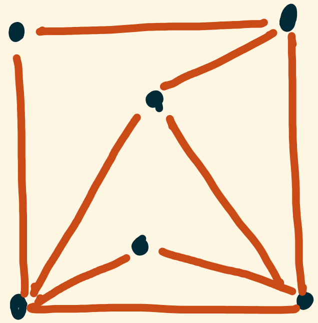


[Every vertex has a unique reflex angle]

POINTED PSEUDO-TRIANGULATIONS







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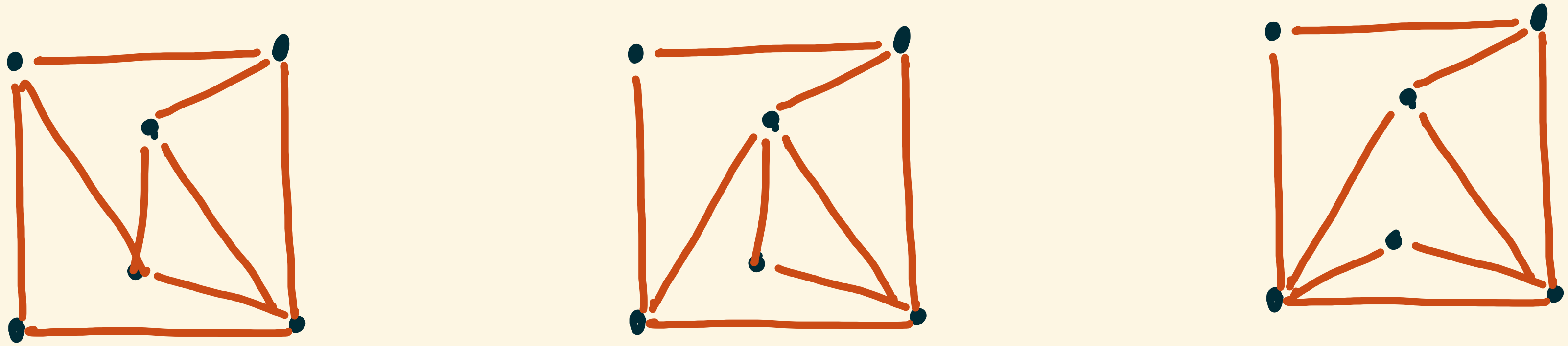
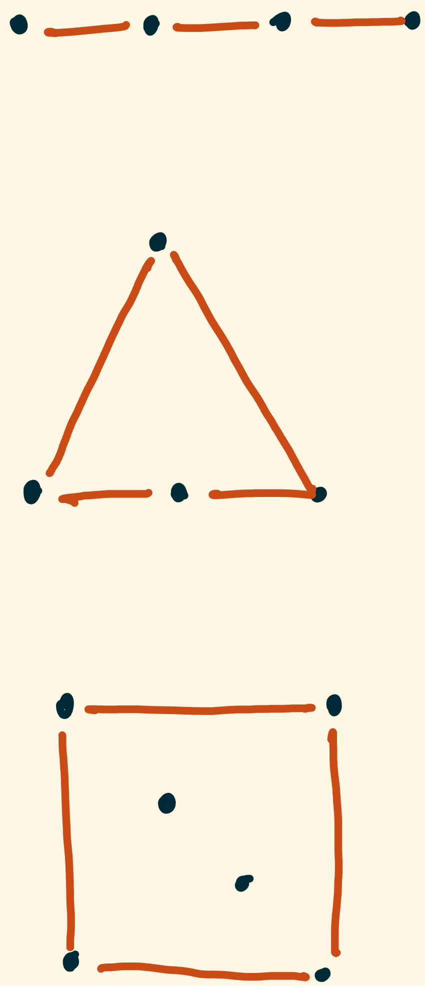


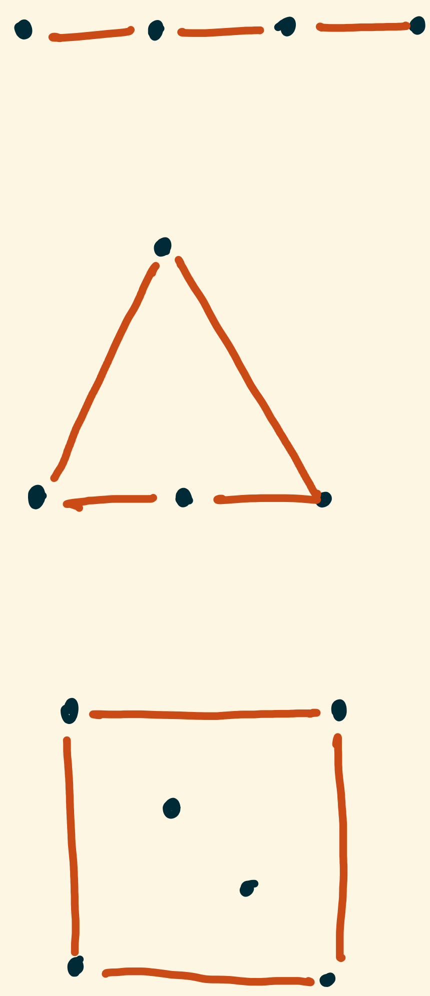
Facts [Streinu, 2000]

- Any ppt of n points has (2n-3) edges
- Any internal edge is uniquely flippable.
- The flip graph is connected

NON-GENERIC CONFIGURATIONS

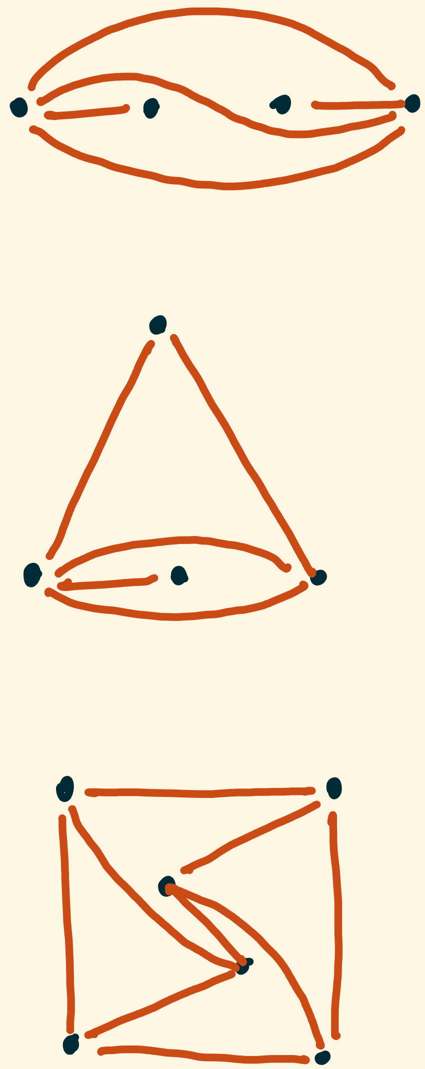


NON-GENERIC CONFIGURATIONS



Q: How to define triangulations/ppts for points not in general position, so that they enjoy the same properties?

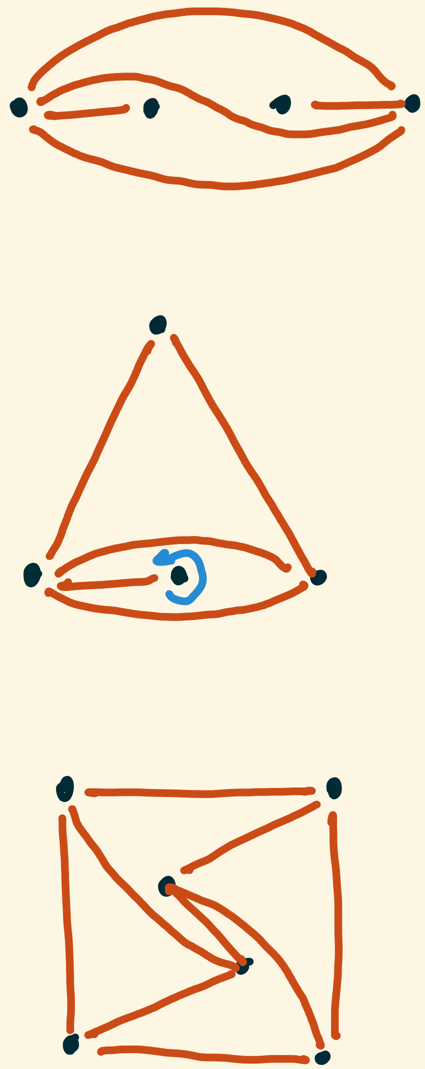
NON-GENERIC CONFIGURATIONS



Answer [B-Deopurkar-Licata]

- Instead of straight segments, consider "strings" that are "pulled tight" around points.

NON-GENERIC CONFIGURATIONS

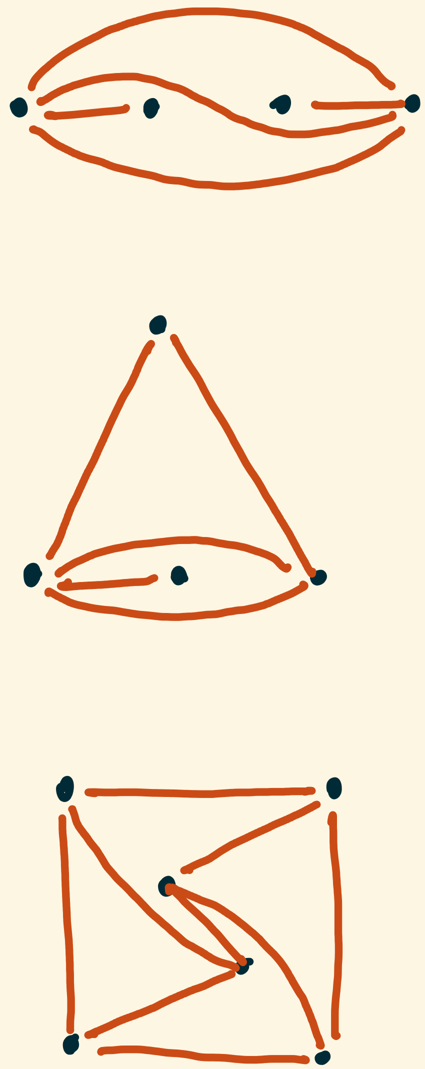


Answer [B-Deopurkar-Licata]

- Instead of straight segments, consider "strings" that are "pulled tight" around points.

- Now angles may take the values 0, π, and 2π!

NON-GENERIC CONFIGURATIONS

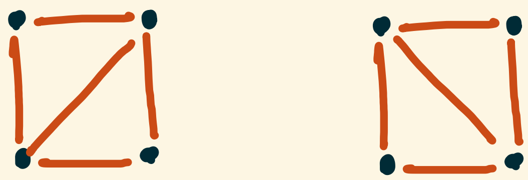


Proposition [B-Deopurkar-Licata]

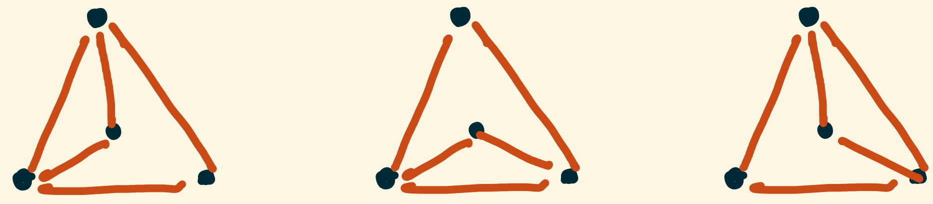
- Every ppt has $2n-3$ edges
- Every internal edge is uniquely flippable.
- The flip graph is connected.

A SIMPLICIAL COMPLEX

Maximal simplices:



or



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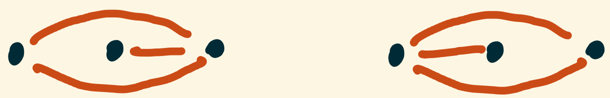

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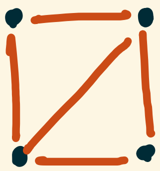
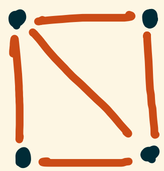
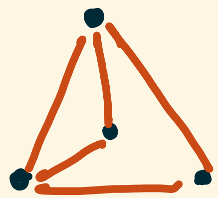
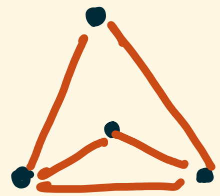
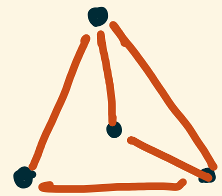
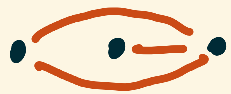
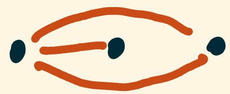
  or   

or  

Fact : This forms a convex polytope of dim $2n-4$

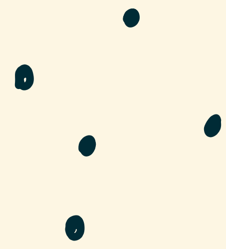
A CATEGORICAL REINTERPRETATION

Category $\mathcal{C}_n$ = 2CY category for A_n quiver

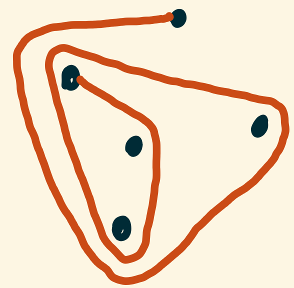
A CATEGORICAL REINTERPRETATION

Dictionary

Bridgeland stability condition on $\mathcal{C}_n$ $\longleftrightarrow$ Labelled point configuration

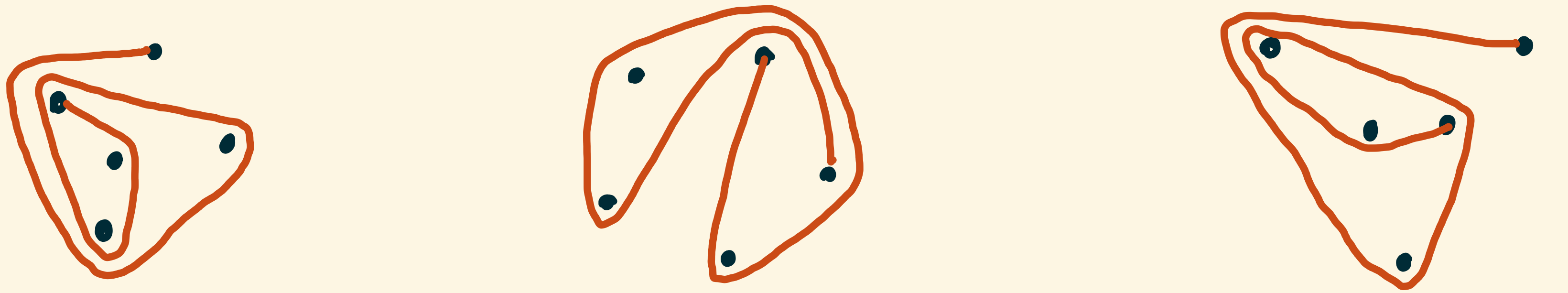


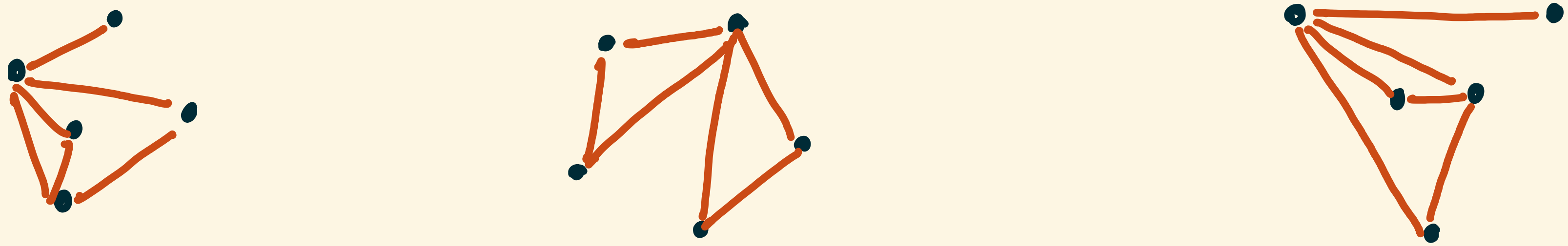
Spherical object of $\mathcal{C}_n$ $\longleftrightarrow$ Non-crossing curve



A CATEGORICAL REINTERPRETATION

Fact: Non-crossing curves pull tight to (subsets of) ppts \ external edge.





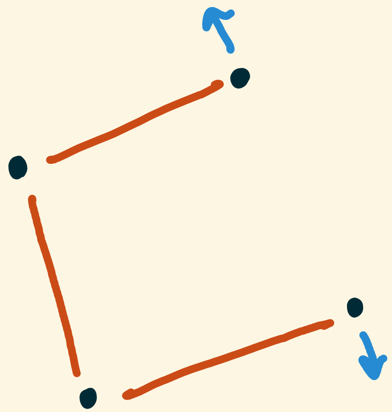
A CATEGORICAL REINTERPRETATION

Theorem [B-Deopurkar-Licata]

For any (labelled) configuration of n points, the spherical objects of $\mathcal{C}_n$ are naturally in bijection with a dense subset of the boundary of the polytope of ppts.

THANK YOU !

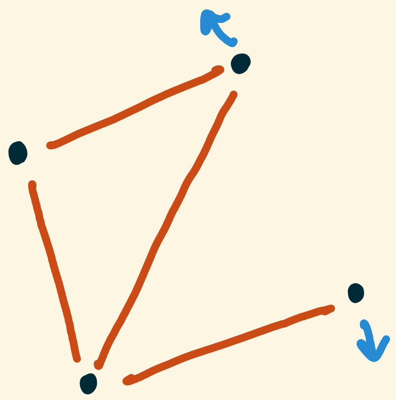
EXPANSIVE RIGID MOTIONS



Expansive motion of a point & rod configuration

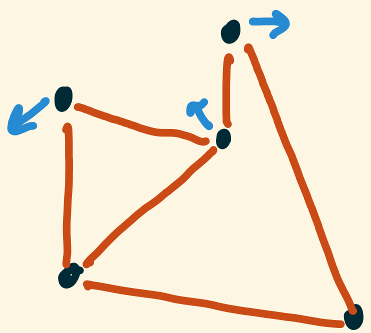
[preserves rod lengths, weakly increases distance between any two vertices.]

EXPANSIVE RIGID MOTIONS



Theorem [Streinu]

Configurations with a unique non-trivial expansive motion = ppts with an external edge removed.



Theorem [BDL]

Same for non-generic configurations