

\* Course admin

- Assignments, marks, etc on Wattle. Quiz submission TBA. video recordings
- Discussion forum on Zulip.
- Workshops start in Week 3, signup on Wattle.
- Office hour from 11-12 Wed & 11-12 Thurs or by appointment. - Details on zulip, subject to change
- Notes available at <https://asilata.github.io/GGM>

\* Assessment

- 45% final exam
- 20% mid-semester exam
- 30% workshop quizzes New! "Practice assignments" each week + in-workshop quizzes
- 5% workshop participation

\* Course outline

- Basic language & foundations
- Graphs & posets
- Finite automata + regular languages (machines)
- Games (combinatorial game theory)

\* What is the point of this course?

Learning abstraction!

- How to forget unimportant details & remember the relevant things
- Techniques to model various situations mathematically.

I. Some foundations & Language for the course

\* Sets

Informally, a set is an unordered collection of objects.

Examples:  $\{1, 2, 4\}$  or  $\{a, b, c\}$  or  $\{x \in \mathbb{N} \mid x \text{ is even}\}$   
↑ natural numbers

[Formally: Zermelo-Fraenkel set theory ...]

Two sets are equal if and only if they have the same elements.

$A=B$  means  $x \in A$  iff  $x \in B$ .

\* Some constructions

- (1) Subset:  $A \subseteq B$  (or  $A \subset B$ ) if every  $x \in A$  is in  $B$
- (2) Superset:  $B \supseteq A$  (or  $B \supset A$ ) if every  $x \in A$  is in  $B$ , i.e.  $A \subseteq B$ .
- (3) Empty set:  $\emptyset$  is the unique set with no elements.
- (4) Union:  $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- (5) Intersection:  $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- (6) Power set:  $P(A) = \{S \mid S \subset A\}$  sets can have other sets as elements!
- (7) Cartesian product:  
 $A \times B = \{(a, b) \mid a \in A, b \in B\}$   
↑ ordered pair

Examples: -  $P(\{1, 3\}) = \{\{1, 3\}, \emptyset\}$

-  $\{1, 2\} \times \{2, 3, 4\} = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\}$

-  $P(\emptyset) = \{\emptyset\}$

-  $\emptyset \times \{5, 6\} = \emptyset$

## \* Relations :

Informally : A way/property that links objects together.

E.g. - Canberra is related to Melbourne b/c they are both cities in Au.

- Sydney is related to NSW b/c it's a city in NSW.
- Echidnas are related to platypuses b/c they've both monotremes.

Let's try to formalize :

Defn : A relation  $R$  on sets  $S$  and  $T$  is a subset  $R \subseteq S \times T$ .

(More precisely, this is a binary relation.)

An  $n$ -ary relation would be a subset of  $S_1 \times S_2 \times \dots \times S_n$ .

\* If  $(a, b) \in R$ , we say  $(a, b)$  satisfies  $R$ , or  $a R b$ .

\* A (binary) relation on a set  $S$  is just a subset  $R \subseteq S \times S$ .  $\leftarrow$  we'll focus on this.

An  $n$ -ary relation on  $S$  is a subset  $R \subseteq \underbrace{S \times \dots \times S}_{n \text{ times}}$ .

\* In general,  $R$  can be quite arbitrary.

Later we'll see examples of special properties

that some relations have

## \* Functions -

A function is a special kind of relation, satisfying the following.

Suppose  $R \subseteq S \times T$ . Then  $R$  is a function if whenever  $(a, b) \in R$ , and  $(a, c) \in R$  then  $b = c$ .

(There is at most one pair in  $R$  with a given 1<sup>st</sup> coordinate. This tells you that  $b$  is determined from  $a$ , so we'll usually write  $b = f(a)$ .)

We say that:

\* the domain of  $f = \{a \in A \mid \text{there is some } (a, b) \in R\}$

\* the codomain of  $f = \{b \in B \mid \text{there is some } (a, b) \in R\}$   
(range)

\* Non-example :  $\{(a, b) \in \mathbb{N} \times \mathbb{N} \mid a + b \text{ is even}\}$ .

this is not a function: b/c  $(2, 4) \in R$ ,  $(2, 0) \in R$ .

\* Example :  $\{(a, b) \in \mathbb{N} \times \mathbb{N} \mid b = a^2\}$ .