MATH 2301 : Games, graphs & machines 29/07/2020

* Course admin
  - Assignments, marks, video recordings etc on Wattle. Quiz submission TBA.
  - Discussion forum on Zulip.
  - Workshops start in Week 3, signup on Wattle.
  - Office hour from 11-12 Wed & 11-12 Thurs or by appointment. — Details on zulip, subject to change
  - Notes available at https://asilata.github.io/GGM

* Assessment
  - 45% final exam
  - 20% mid-semester exam
  - 30% workshop quizzes ⇝ New! "Practice assignments" each week + in-workshop quizzes
  - 5% workshop participation

* Course outline
  - Basic language & foundations
  - Graphs & posets
  - Finite automata + regular languages (machines)
  - Games (combinatorial game theory)

* What is the point of this course?

  Learning abstraction!

  - How to forget unimportant details & remember the relevant things
  - Techniques to model various situations mathematically.

I. Some foundations ← Language for the course

* Sets

Informally, a set is an unordered collection of objects.

Examples: $\{1,2,4\}$ or $\{a,b,c\}$ or $\{x \in \mathbb{N} \mid x \text{ is even}\}$
natural numbers

[Formally: Zermelo-Fraenkel set theory ...]

Two sets are equal if and only if they have the same elements.

$A = B$ means $x \in A$ iff $x \in B$.

* Some constructions

(1) Subset: $A \subseteq B$ (or $A \subset B$) if every $x \in A$ is in B

(2) Superset: $B \supseteq A$ (or $B \supset A$) if every $x \in A$ is in B, i.e. $A \subseteq B$.

(3) Empty set: $\emptyset$ is the unique set with no elements.

(4) Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

(5) Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

(6) Power set: $P(A) = \{S \mid S \subset A\}$ ← Sets can have other sets as elements!

(7) Cartesian product:
$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$
ordered pair

Examples:
- $P(\{1\}) = \{\{1\}, \emptyset\}$
- $\{1,2\} \times \{2,3,4\} = \{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4)\}$
- $P(\emptyset) = \{\emptyset\}$
- $\emptyset \times \{5,6\} = \emptyset$.

* Relations :

Informally : A way/property that links objects together.

E.g. - Canberra is related to Melbourne b/c they are both cities in Au.

- Sydney is related to NSW b/c it's a city in NSW.
- Echidnas are related to platypuses b/c they're both monotremes.

Let's try to formalize :

Defn : A relation R on sets S and T is a subset $R \subseteq S \times T$.

(More precisely, this is a binary relation.

An n-ary relation would be a subset of $S_1 \times S_2 \times \cdots \times S_n$.)

* If $(a,b) \in R$, we say (a,b) satisfies R, or aRb.

* A (binary) relation on a set S is just a subset $R \subseteq S \times S$. ← we'll focus on this.

An n-ary relation on S is a subset $R \subseteq \underbrace{S \times \cdots \times S}_{n \text{ times}}$

* In general, R can be quite arbitrary.

Later we'll see examples of special properties that some relations have

* Functions.

A function is a special kind of relation, satisfying the following.

Suppose $R \subseteq S \times T$. Then R is a function if whenever $(a,b) \in R$, and $(a,c) \in R$ then $b = c$.

(There is at most one pair in R with a given 1^{st} coordinate. This tells you that b is determined from a, so we'll usually write $b = f(a)$.)

We say that:

* the domain of $f = \{a \in A \mid \text{there is some } (a,b) \in R\}$
* the codomain (range) of $f = \{b \in B \mid \text{there is some } (a,b) \in R\}$
* Non-example : $\{(a,b) \in \mathbb{N} \times \mathbb{N} \mid a+b \text{ is even}\}$.

  this is not a function: b/c $(2,4) \in R$, $(2,0) \in R$.
* Example : $\{(a,b) \in \mathbb{N} \times \mathbb{N} \mid b = a^2\}$.