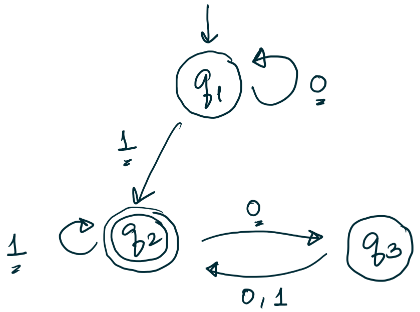
MATH 2301 01 Oct 2020



* A "state diagram" of a DFA. (deterministic finite automaton)

Start state q_1

q_2 is the only accept state

$\Sigma = \{0,1\}$

$s = 0110$ (rejected)

* Formal/mathematical definition of a DFA:

Defn: A DFA consists of 5 pieces of data:

(1) An alphabet Σ

(2) A set Q of "states"

(3) A "start state" $q_1 \in Q$.

(4) A set $A \subseteq Q$ of "accept states"

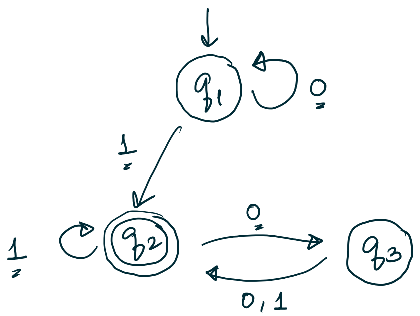
(5) A "transition function" $\delta : Q \times \Sigma \to Q$.

This means that every state has an outgoing arrow for each letter (not necessarily distinct.)

In previous example, $\Sigma = \{0,1\}$

$Q = \{q_1, q_2, q_3\}$, $A = \{q_2\}$

Let's try to write $\delta : Q \times \Sigma \to Q$



$\delta(q_1, 0) = q_1$

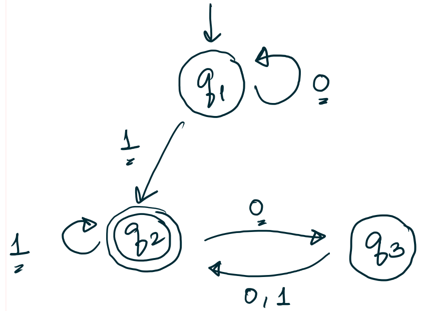
$\delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_3$

$\delta(q_2, 1) = q_2$

$\delta(q_3, 0) = q_2$

$\delta(q_3, 1) = q_2$.



* Given a DFA M, the language of M, denoted $L(M)$
$= \{ s \in \Sigma^* \mid s \text{ is accepted by } M \}$

[Check: If $r = 0^*1(1 | 00 | 01)^*$ then $L(r) = L(M)$ for this example]

We'll want to prove the following theorem:

Thm: A language L is a regular language if and only if there is some DFA M, such that $L(M) = L$.

Exercise: Can you give a description in words of the language of the previous DFA?

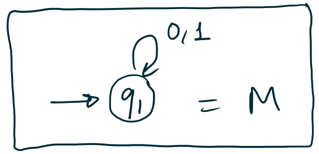
---

* For simplicity, assume usually that $\Sigma = \{0,1\}$.

Regular expressions $\longrightarrow$ DFAs ?

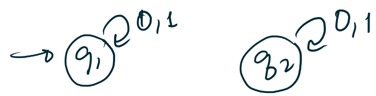
① $\boxed{r = \phi}$

$L(r) = \phi$

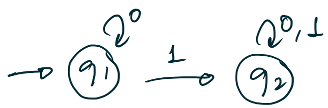


$L(M) = \phi$.

Here's another example:



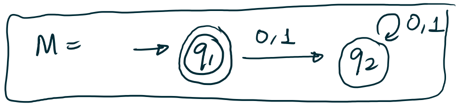
← this is funny, b/c you never reach q_2, but it is technically allowed.



[lots of options!]

② $\boxed{r = \varepsilon}$

$L(r) = \{\varepsilon\}$



$L(M) = \{\varepsilon\}$.

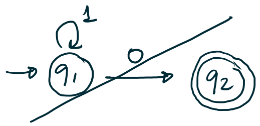
[Again, there are other options for M.]

③ $r = a$ for some $a \in \Sigma$

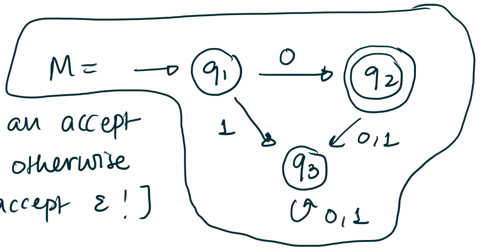
Let's take $\boxed{r = 0}$

$L(r) = \{0\}$

M =



doesn't work: accepts 10



[q_1 not an accept state, otherwise we'd accept ε!]

$L(M) = \{0\}$

<u>Rmk</u>: Make similar constructions for other letters of the alphabet.

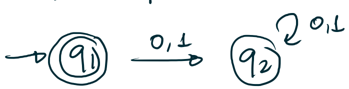
④ $r = r_1 \mid r_2$ where r_1, r_2 are valid regular expressions

Let's assume that we have DFAs M_1 & M_2 such that $L(M_1) = L(r_1)$, $L(M_2) = L(r_2)$. We'll try to construct M such that $L(M) = L(r)$.

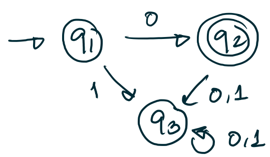
We'll use the formal definition of DFA to help us here.

<u>Example</u>: $r = \varepsilon \mid 0$

$M_1 =$



$M_2 =$



<u>Heuristic</u>: M should simultaneously simulate both M_1 & M_2, and accept if <u>one</u> of them accepts.

<u>Idea</u>: Use a "product" automaton.