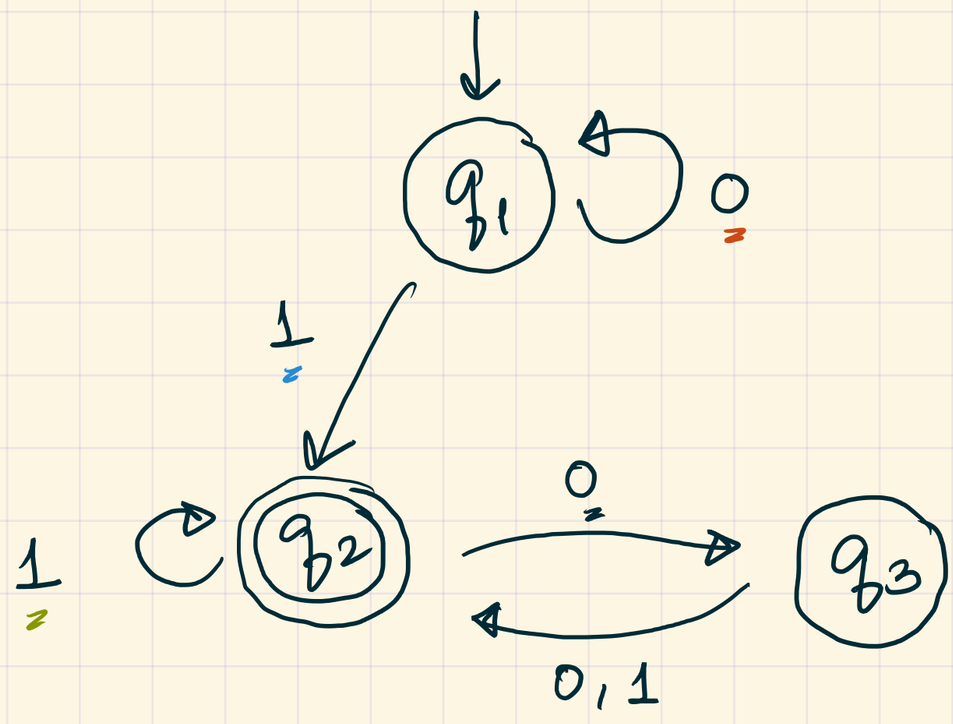
MATH 2301 — 01 Oct 2020



* A "state diagram" of a DFA.

(deterministic finite automaton)

Start state q_1

q_2 is the only accept state

$\Sigma = \{0, 1\}$

$S = \underset{z}{0}\ \underset{z}{1}\ \underset{z}{1}\ \underset{z}{0}$ (rejected)

* Formal/mathematical definition of a DFA:

Defn: A DFA consists of 5 pieces of data:

(1) An alphabet Σ

(2) A set Q of "states"

(3) A "start state" $q_1 \in Q$.

(4) A set $A \subseteq Q$ of "accept states"

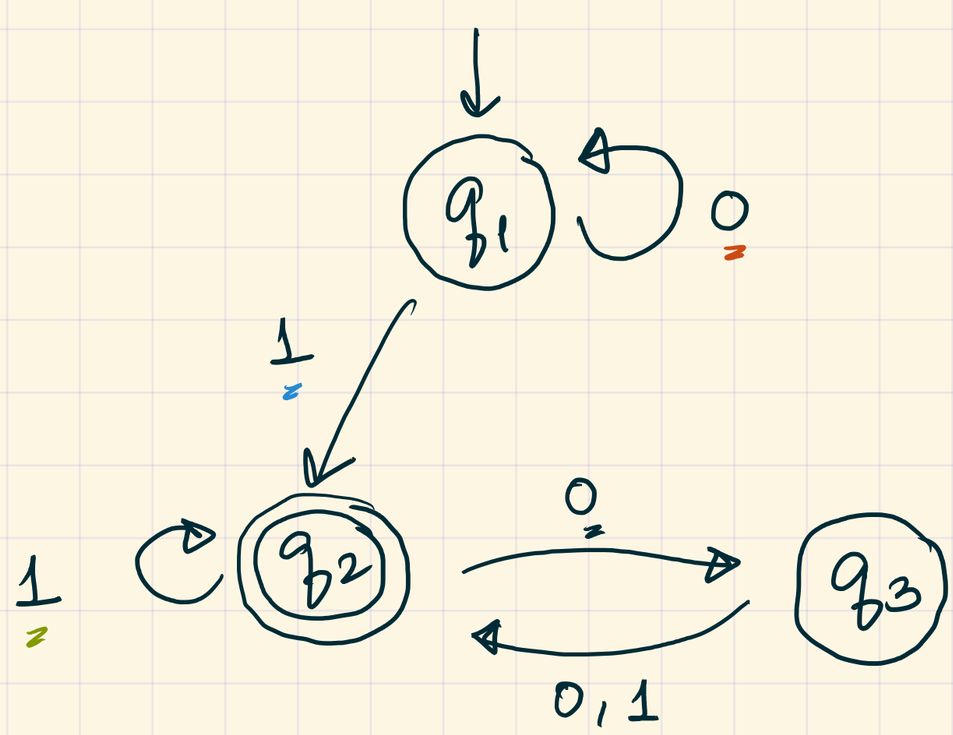
(5) A "transition function" $\delta : Q \times \Sigma \to Q$.

↑ This means that every state has an outgoing arrow for each letter (not necessarily distinct.)

In previous example, $\Sigma = \{0, 1\}$

$Q = \{q_1, q_2, q_3\}$, $A = \{q_2\}$

Let's try to write $\delta : Q \times \Sigma \to Q$



$\delta(q_1, 0) = q_1$

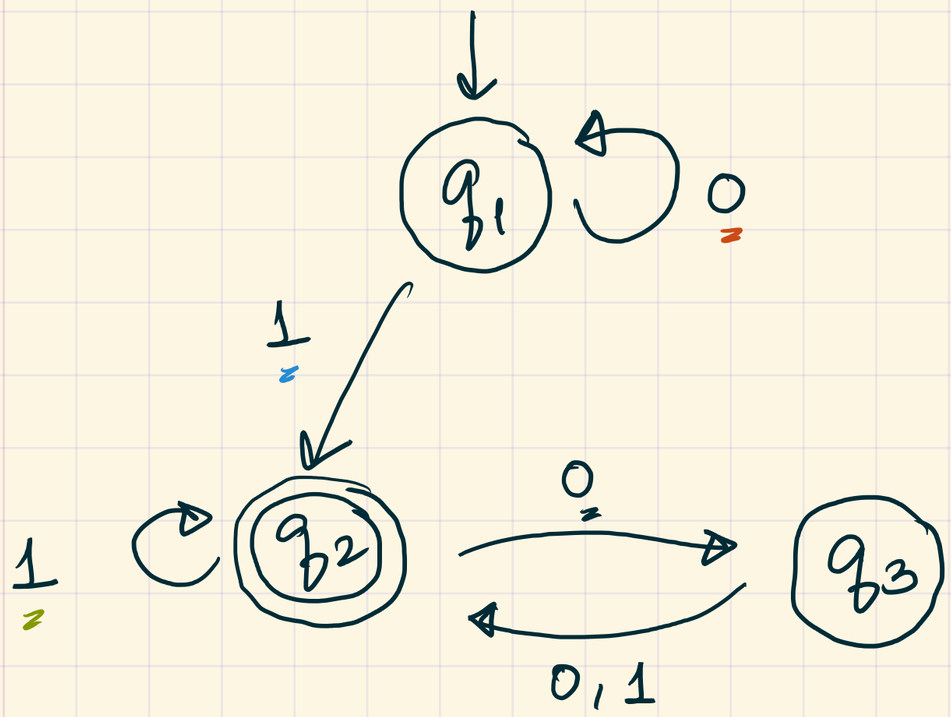
$\delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_3$

$\delta(q_2, 1) = q_2$

$\delta(q_3, 0) = q_2$

$\delta(q_3, 1) = q_2$.



* Given a DFA M, the language of M, denoted $L(M)$

$= \{ s \in \Sigma^* \mid s \text{ is accepted by } M \}$

[Check: If $r = 0^*1(1|001)^*$ then $L(r) = L(M)$ for this example]

We'll want to prove the following theorem:

Thm: A language L is a regular language if and only if there is some DFA M, such that $L(M) = L$.

Exercise: Can you give a description in words of the language of the previous DFA?

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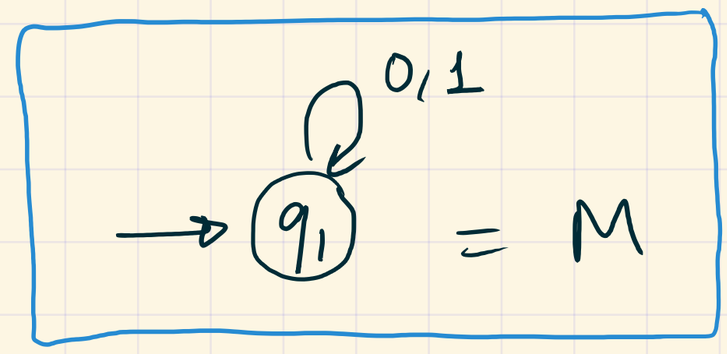
* For simplicity, assume usually that $\Sigma = \{0, 1\}$.

Regular expressions ⟶ DFAs?

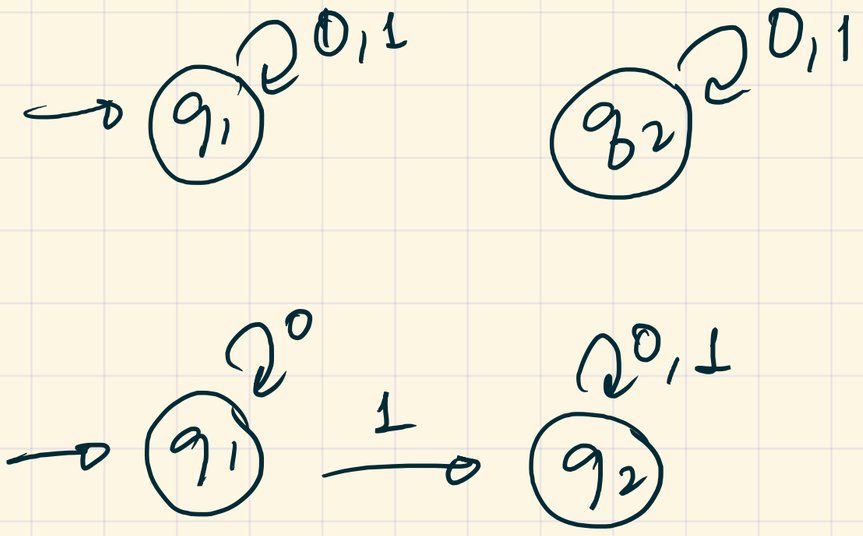
① $r = \phi$

$L(r) = \phi$



$L(M) = \phi$.

Here's another example:

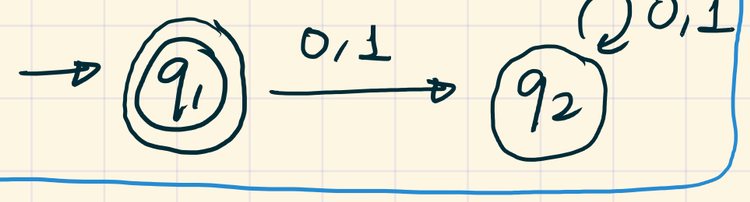


← this is funny, b/c you never reach q_2, but it is technically allowed.

[lots of options!]

② $r = \varepsilon$

$L(r) = \{\varepsilon\}$

$M =$ 

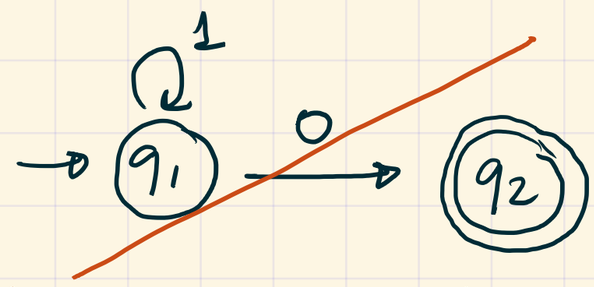
$L(M) = \{\varepsilon\}$.

[Again, there are other options for M.]

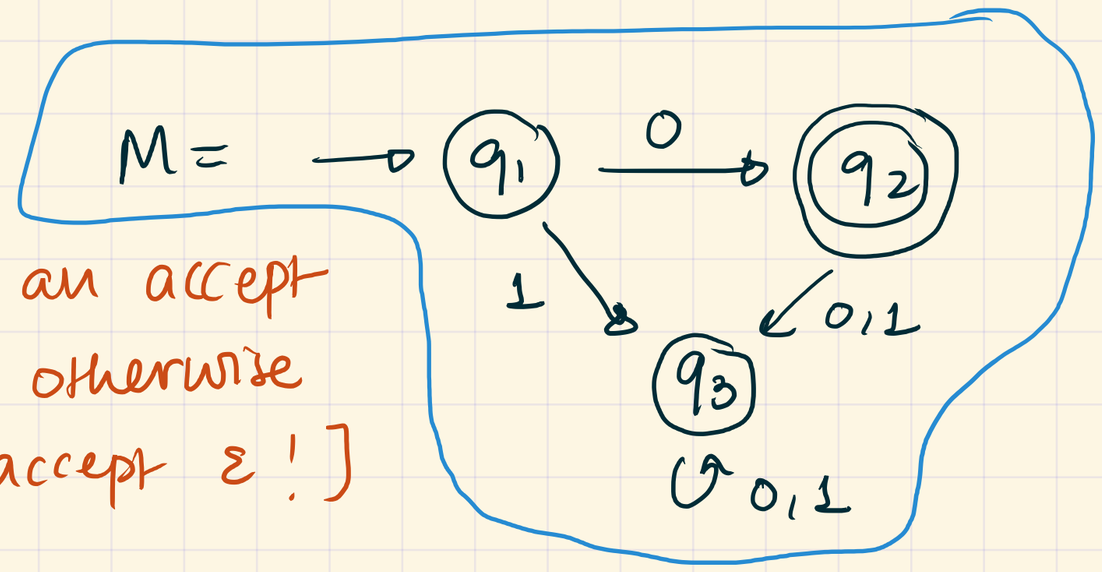
③ $r = a$ for some $a \in \Sigma$

Let's take $r = 0$

$L(r) = \{0\}$

$M =$ 

doesn't work: accepts 10

$M =$ 

[q_1 not an accept state, otherwise we'd accept ε!]

$L(M) = \{0\}$

Rmk: Make similar constructions for other letters of the alphabet.

④ $r = r_1 \mid r_2$

where

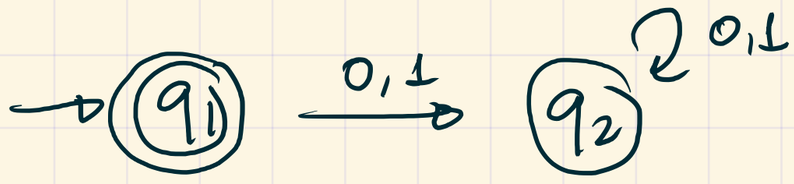
r_1, r_2 are valid regular expressions

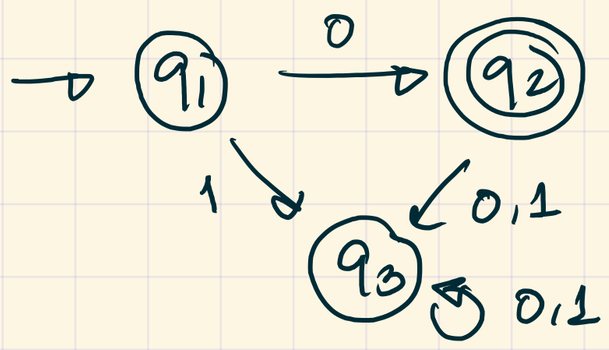
Let's assume that we have DFAs M_1 & M_2 such that

$L(M_1) = L(r_1)$, $L(M_2) = L(r_2)$.

We'll try to construct M such that $L(M) = L(r)$.

Example: $r = \varepsilon \mid 0$

$M_1 =$ 

$M_2 =$ 

Heuristic: M should simultaneously simulate both M_1 & M_2, and accept if one of them accepts.

Idea: Use a "product" automaton.