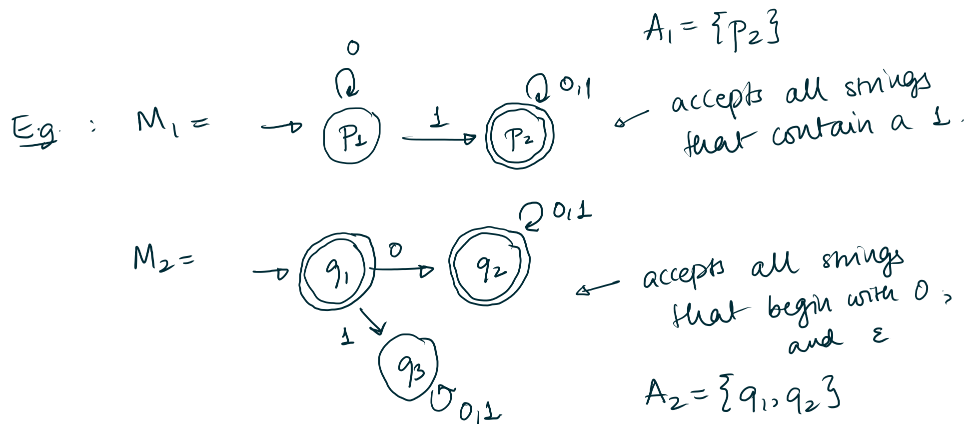
MATH 2301 02 October 2020

* Recap : We've built DFAs that recognise the languages $\emptyset$ (1), $\{\varepsilon\}$ (2), and $\{a\}$ (3) for any $a \in \Sigma$

Next, we're trying to build a DFA M such that $L(M) = L(r_1|r_2) = L(r_1) \cup L(r_2)$ given DFAs M_1 & M_2 such that $L(M_1) = L(r_1)$, $L(M_2) = L(r_2)$

Idea : Construct M to simulate both M_1 & M_2 on any given input, and to accept if any one accepts.

Let :
* P, Q be the states of M_1, M_2 resp.
* p_1, q_1 be the start states of M_1, M_2 resp.
* A_1, A_2 be the sets of accept states of M_1, M_2 resp
* $\delta_1 : P \times \Sigma \to P$, $\delta_2 : Q \times \Sigma \to Q$ transition fns of M_1, M_2 resp.



Construct M as the "product automaton".

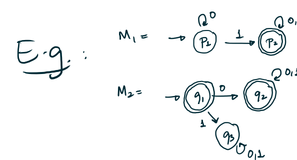
* The states of M : $P \times Q$.
The state (p,q) represents that we're at p in M_1 and q in M_2.

* Start state : (p_1, q_1), representing that we start both in M_1 & in M_2

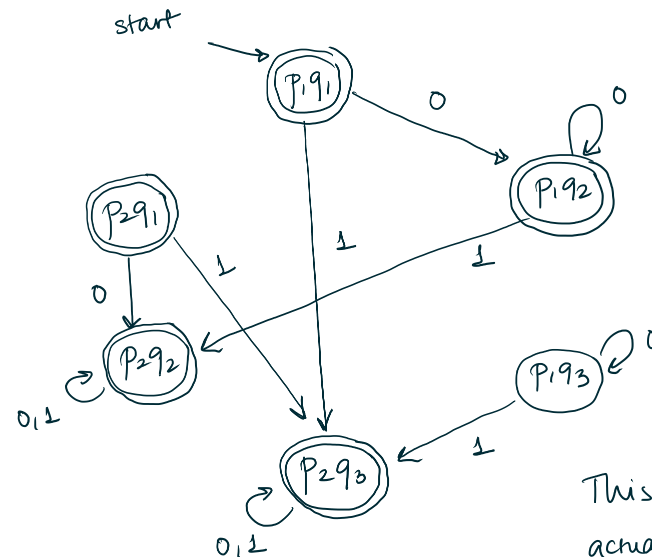
* Transition function :
$\delta : (P \times Q) \times \Sigma \to (P \times Q)$
$(p,q), a \mapsto (\delta_1(p,a), \delta_2(q_2,a))$.

* Accept states :

E.g. :



We have 6 states in M :
(p_1,q_1), (p_1,q_2), (p_1,q_3)
(p_2,q_1), (p_2,q_2), (p_2,q_3)



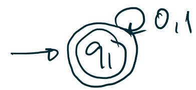
For transitions, follow M_1 for the first coordinate and M_2 for the second coordinate.

(p,q) is accepting if either $p \in A_1$ or $q \in A_2$

This machine M actually accepts everything!

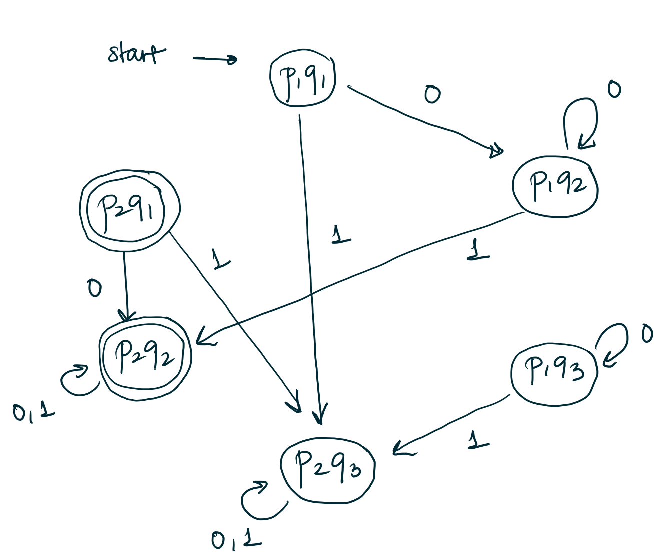
* M will exactly accept $L(M_1) \cup L(M_2)$, but may not be the efficient machine to do so.

Here is another machine that accepts every string:



Here is something else you can do with a product automaton: you can change the accepting states to produce a machine M' that accepts $L(M_1) \cap L(M_2)$

[In example: strings that begin with a 0 and contain at least one 1]



(p,q) is accepting if $p \in A_1$ and $q \in A_2$, otherwise it's the same product construction.

Given $r = r_1 | r_2$ & DFAs M_1, M_2 st $L(M_i) = L(r_i)$, we have now constructed M, such that $L(M) = L(r)$. (4th constructor for regex)

The remaining two are: ⑤ concatenation: $r = r_1 r_2$, and

⑥ star : $r = (r_1)^*$

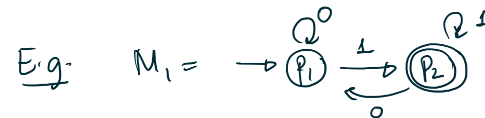
⑤: Assume we have M_1, M_2 such that $L(r_i) = L(M_i)$

Want to build M, such that $L(M) = L(r_1 r_2)$.

Problem: Given $w \in \Sigma^*$ that matches $r_1 r_2$, we know that $w = xy$ such that $x \in L(r_1)$, $y \in L(r_2)$.

But we don't know the lengths of x & y, and in fact, there could be more than one way to break w!

Idea: Take M_1



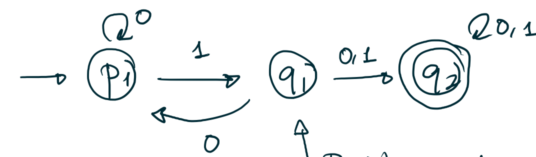
& for each accepting state, we'd like to attach a copy of M_2.

Problem: How do you deal with the outgoing arrows from the accept states of M_1?

E.g. $M_1 =$

$M_2 =$

Suggested procedure produces:



Problem: Two outgoing arrows labelled 0!

* Next week: We'll use non-deterministic finite automata (NFAs) to solve this problem!