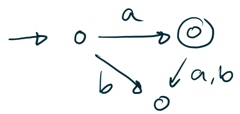
MATH 2301 14 Oct 2020

* Continued from last time:

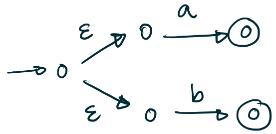
Let $\Sigma = \{a, b\}$

$r = (a|b)^* aba$

NFA for $\{a\}$:  or 

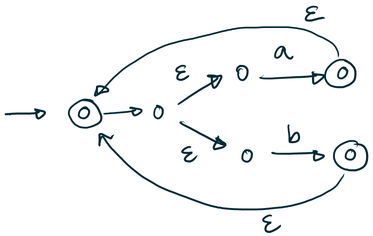
NFA for $\{b\}$: 

NFA for $a|b$



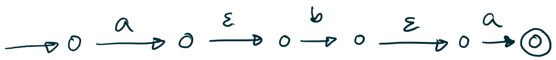
(or product construction of DFAs)

NFA for $(a|b)^*$:



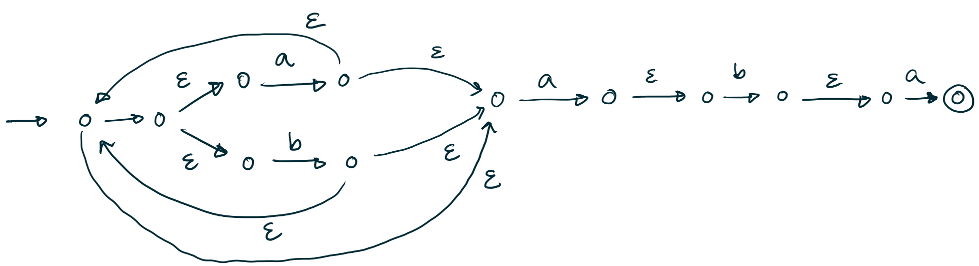
[actually, we could have used  but let's continue with what the procedure produces]

NFA to recognise aba, ie. the language $\{aba\}$



[using concatenation procedure]

NFA for $(a|b)^* aba$:



Summary so far:

Thm: Let $L \subseteq \Sigma^*$ be a language. The following are equivalent:

(1) L is a regular language

(2) $L = L(r)$ for some valid regex r

(3) $L = L(M_1)$ for some DFA M_1

(4) $L = L(M_2)$ for some NFA M_2.

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* Unfortunately, not all languages are regular. But how can we tell??

* We need some method of proof that can conclusively say that there is no DFA/NFA/regex that gives that language.

Eg. 1) $L_1 = \{w \mid w = 0^n 1^n \text{ for some } n \in \mathbb{N}\}$

Eg. 2) $L_2 = \{w \mid w \text{ has an equal number of 1s \& 0s}\}$

Eg. 3) $L_3 = \{w \mid w \text{ has an equal number of the substrings 01 \& 10}\}$.

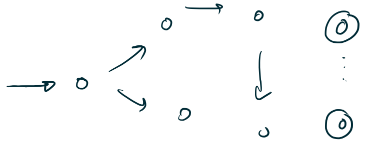
Answer: L_1 & L_2 are not regular, but L_3 is regular.

* Intuition: If recognising your language can't be done with a finite amount of memory, then it isn't regular.

* Rmk : If $|L| < \infty$; ie L has finitely many elements, then L is regular (using a finite number of "|" operations)

[& L is not finite.]

* Suppose L is regular, There is some DFA M such that $L = L(M)$.

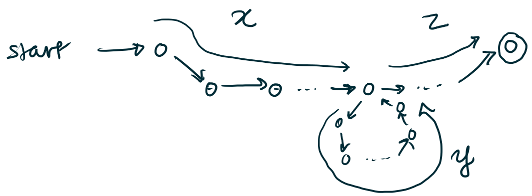


Schematic of M.
It has a finite number of states, say n.

* Because $|L| = \infty$, L has arbitrarily long strings. ie given $N \in \mathbb{N}$, L always has strings of length $> N$.

* What happens if $w \in L$ such that $|w| > n$? (number of states of M.)

$\Rightarrow$ Then w will repeat states in the DFA M, i.e. the path taken by w will have a loop:



schematic of the path taken by w through M.

Write $w = xyz$, where y is the middle portion that begins and ends at the same state.

We know that $|y| \geq 1$. [x, z may have length 0]

* Observe : Because of this, the strings $xyyz$, $xyyyz$, $xyyyyyyyyz$, xz are all in L.

i.e, M will accept all strings of the form $x(y)^*z$