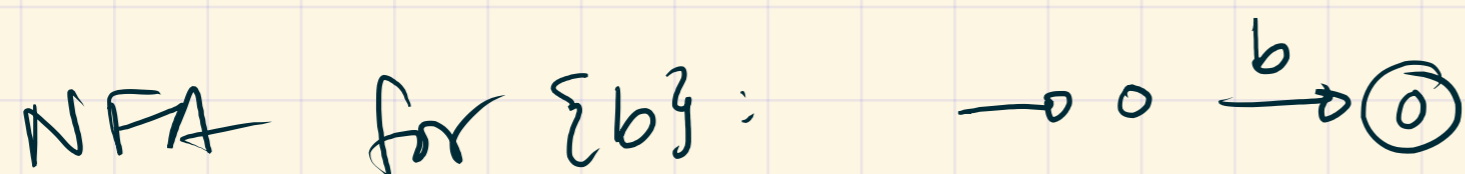
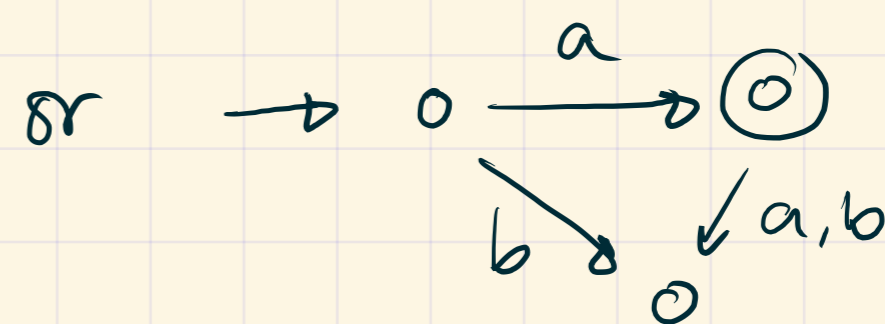


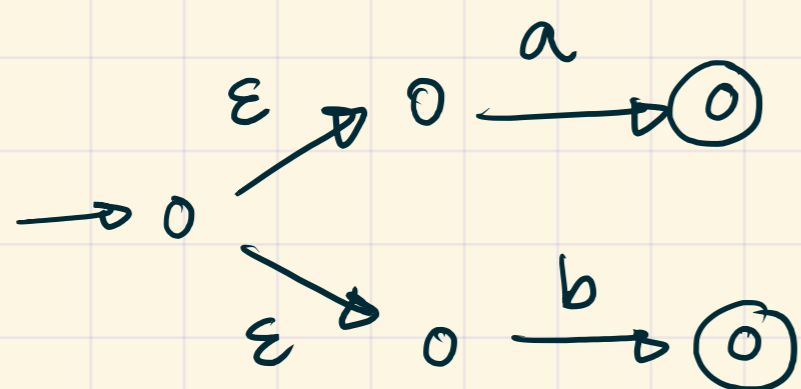
* Continued from last time :

Let $\Sigma = \{a, b\}$

$r = \underbrace{(a|b)^*}_{*} \underbrace{aba}$

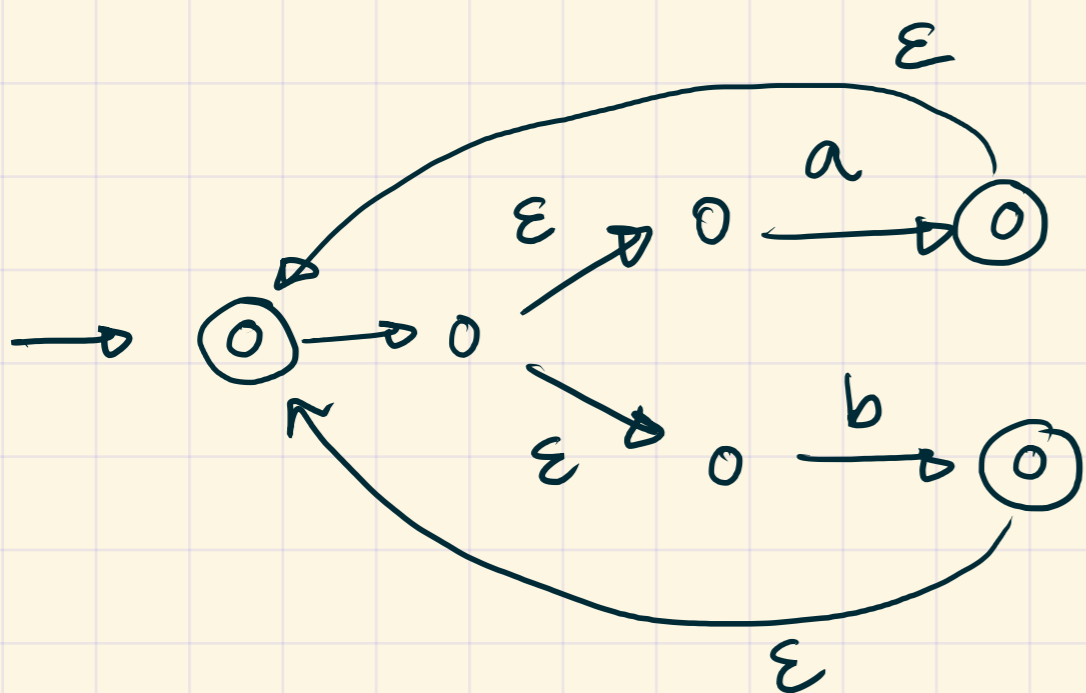


NFA for $a|b$



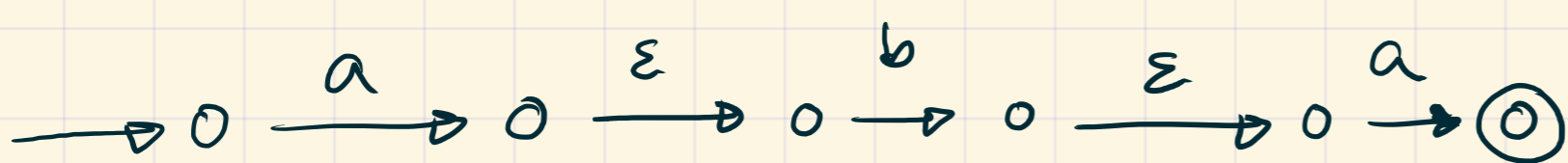
(or product construction of DFAs)

NFA for $(a|b)^*$:



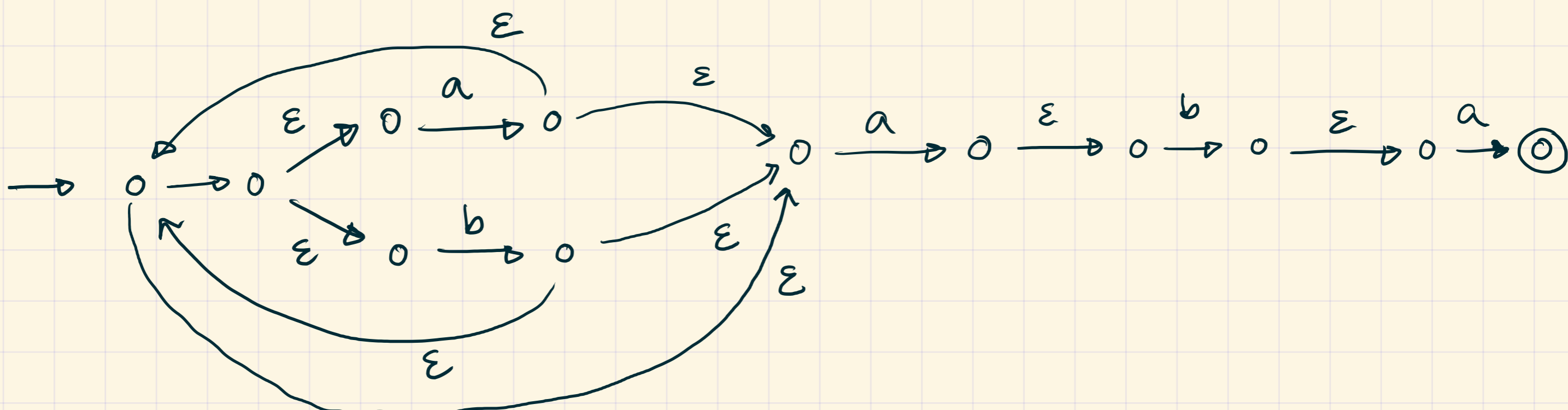
[actually, we could have used $\rightarrow \odot \xrightarrow{a,b} \odot$ but let's continue with what the procedure produces]

NFA to recognise aba , i.e. the language $\{aba\}$



[using concatenation procedure]

NFA for $(a|b)^* aba$:



Summary so far:

Thm: Let $L \subseteq \Sigma^*$ be a language. The following are equivalent:

- (1) L is a regular language
- (2) $L = L(\gamma)$ for some valid regex γ
- (3) $L = L(M_1)$ for some DFA M_1
- (4) $L = L(M_2)$ for some NFA M_2 .

* Unfortunately, not all languages are regular.

But how can we tell??

* We need some method of proof that can conclusively say that there is no DFA/NFA/regex that gives that language.

E.g. 1) $L_1 = \{w \mid w = 0^n 1^n \text{ for some } n \in \mathbb{N}\}$

E.g. 2) $L_2 = \{w \mid w \text{ has an equal number of 1s \& 0s}\}$

E.g. 3) $L_3 = \{w \mid w \text{ has an equal number of the substrings } 01 \text{ \& } 10\}$.

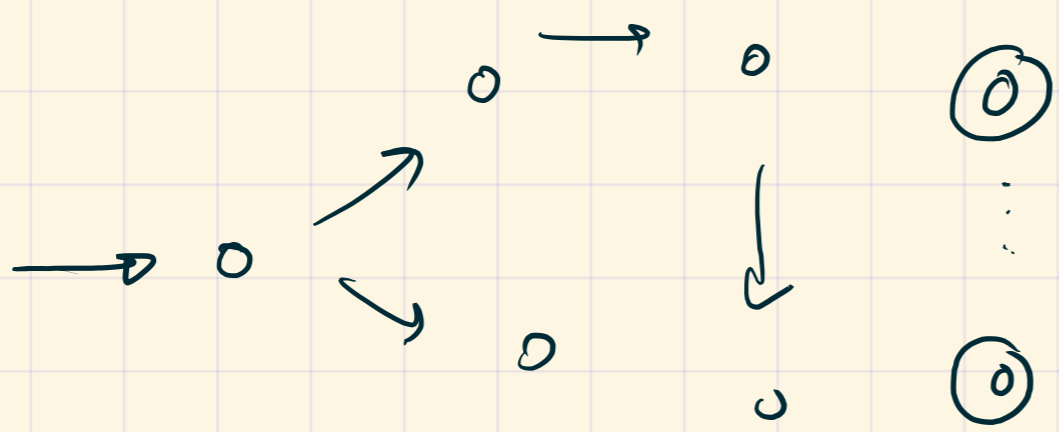
Answer: L_1 & L_2 are not regular, but L_3 is regular.

* Intuition: If recognising your language can't be done with a finite amount of memory, then it isn't regular.

* Remark: If $|L| < \infty$; i.e. L has finitely many elements, then L is regular (using a finite number of "1" operations.)

[$\neq L$ is not finite.]

* Suppose L is regular. There is some DFA M such that $L = L(M)$.

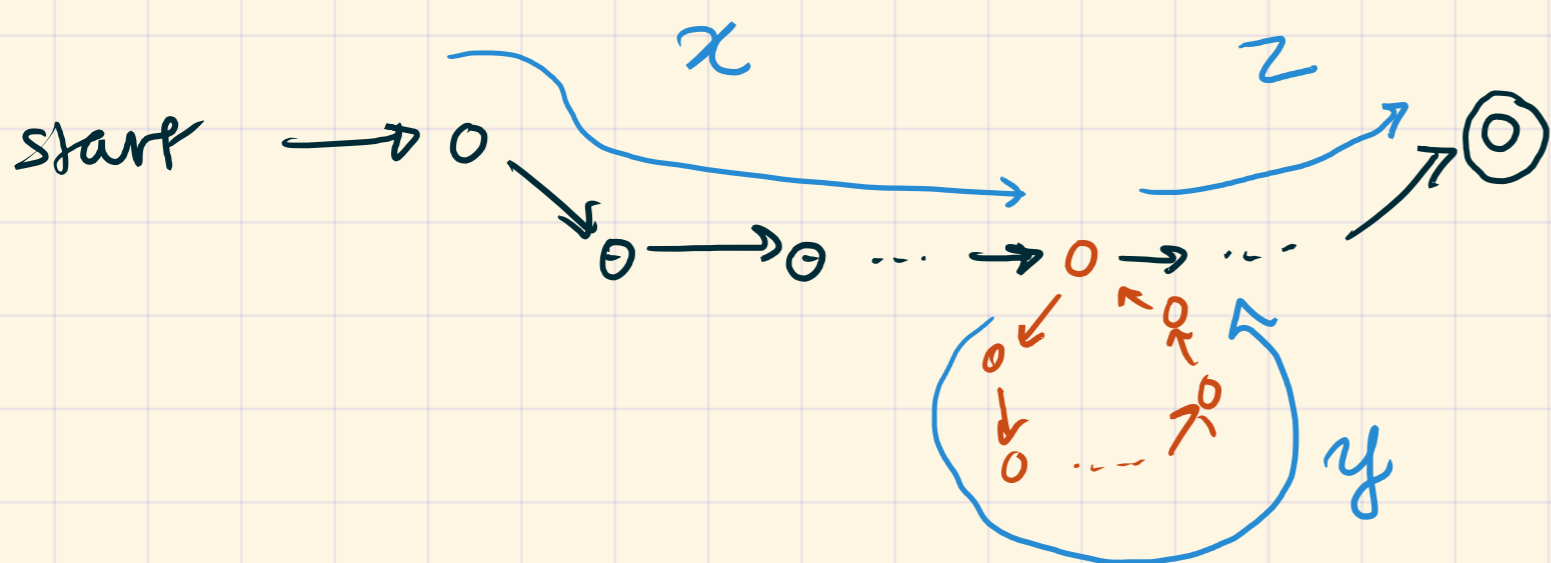


Schematic of M .

It has a finite number of states, say n .

* Because $|L| = \infty$, L has arbitrarily long strings. i.e. given $N \in \mathbb{N}$, L always has strings of length $> N$.

* What happens if $w \in L$ such that $|w| > n$?
 \Rightarrow Then w will repeat states in the DFA M , i.e. the path taken by w will have a loop:



schematic of the path taken by w through M .

Write $w = xyz$, where y is the middle portion that begins and ends at the same state.

We know that $|y| \geq 1$. [x, z may have length 0]

* Observe: Because of this, the strings

$xyyz$, $xyyyz$, $xyyyyyyyz$, xz are all in L .

i.e, M will accept all strings of the form $x(y)^*z$
