

* Impartial combinatorial games:

- Two players; each player moves in turn.
- Each player moves from one "game state" to another, according to some rules
- The rules of a valid move depend only on the game state and not on the player.
(so games like chess are not in this category.)
- There is no hidden information; all the game state is public to both players.
- The player who does not have a valid move loses.
- The game is finite: someone is guaranteed to lose after a finite number of moves.

* Example: Subtraction games.

$S = \{1, 3, 4\}$ | A move consists of subtracting one of the numbers from S from n , remaining non-negative.

Exercise: Play this game a few times and try to see if you can determine who has a winning strategy!

* Strategic labelling

Each game state will be labelled by either:

P : previous player has a winning strategy (aka "0" position)

N : next player has a winning strategy (aka "1" position)



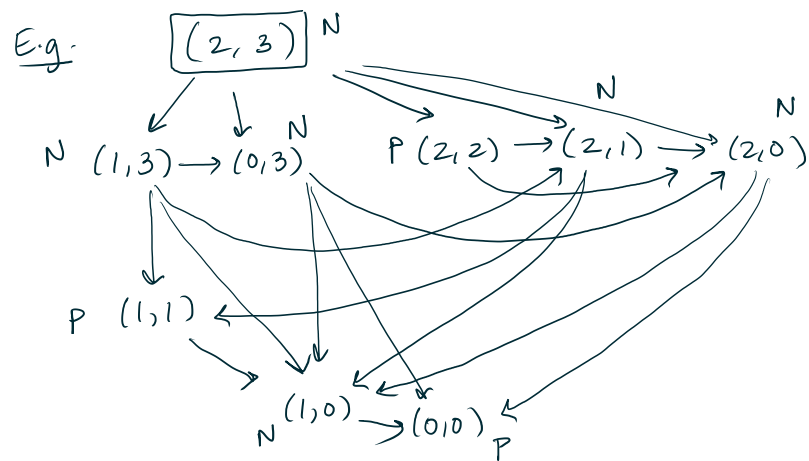
- * If every single move from a given position moves you into an N-position, then label the given position as P.
- * If there is at least one move to a P-position, then label the given position as N.

* In principle, this strategic labelling works for any impartial combinatorial game!

* The game of nim:

- A game position consists of some number of piles of berries.
- A valid move consists of eating any ^{positive} number of berries from a single pile.

- The person who can't make a move loses the game (ie if there are no berries left)



* We'll identify (i,j) with (j,i) .

* This labelling tells us that $(2,3)$ is an N-position
[1st player can win]

* It tells us how to win: always try to move to a P-position!

* But this is cumbersome.