MATH 2301 21 October 2020

* Impartial combinatorial games

- Two players; each player moves in turn.
- Each player moves from one "game state" to another, according to some rules
- The rules of a valid move depend only on the game state and not on the player.

  (so games like chess are not in this category)
- There is no hidden information; all the game state is public to both players
- The player who does not have a valid move loses.
- The game is finite: someone is guaranteed to lose after a finite number of moves.

* Example: Subtraction games

$S = \{1, 3, 4\}$

Let $n = 10$

A move consists of subtracting one of the numbers from S from n, remaining non-negative.

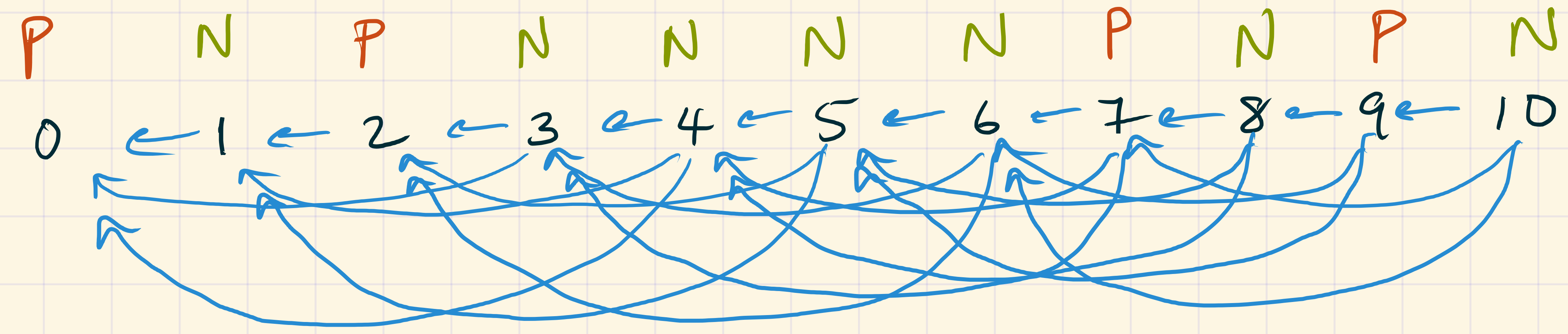
Exercise: Play this game a few times and try to see if you can determine who has a winning strategy!

* Strategic labelling

Each game state will be labelled by either:

P : previous player has a winning strategy (aka "0" position)

N : next player has a winning strategy. (aka "1" position)



* If every single move from a given position moves you into an N-position, the label the given position as P.

* If there is at least one move to a P-position, then label the given position as N.

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* In principle, this strategic labelling works for any impartial combinatorial game!

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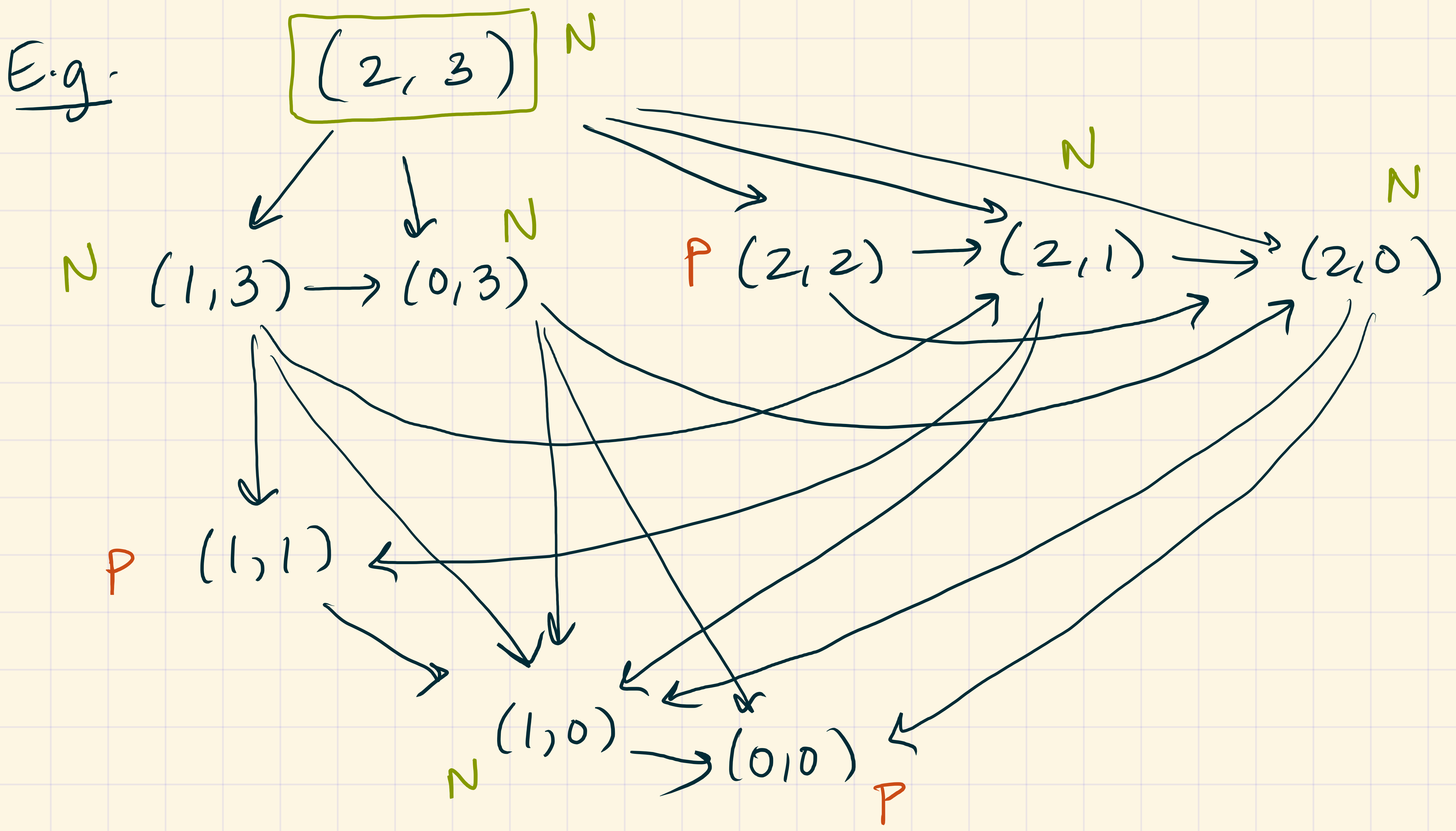
* The game of nim.

- A game position consists of some number of piles of berries.
- A valid move consists of eating any positive number of berries from a single pile.

- The person who can't make a move loses the game. (ie if there are no berries left)

E.g.



* We'll identify (i,j) with (j,i).

* This labelling tells us that $(2,3)$ is an N-position [1^{st} player can win]

* It tells us how to win: always try to move <u>to</u> a P-position!

* But this is cumbersome.