MATH 2301 28 Oct 2020

* Final exam 17 Nov 2020

9am – noon Canberra time

Format: 2 blocks of 1 hr each + 30 min break

Set up on Gradescope (details soon)

Invigilated on Zoom.

Syllabus: Everything covered in lecture, more content from 2^{nd} half of class.

* More details on Wattle by the end of the week.

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* We discussed a strategy for nim

If heap sizes are $x_1, \dots, x_k$, then we compute the nim-sum or nimber $x_1 \oplus x_2 \oplus \cdots \oplus x_k$

Binary XOR

E.g. With a single heap of size n, the corresponding nimber is just n.

(denoted $*n$, indicating that the operation on these nimbers is $\oplus$ and not usual addition)

If the nimber of a nim position equals $*0 \Rightarrow$ P position

Otherwise $\Rightarrow$ N position, and can always move to a position with nimber $*0$.

E.g. $\{2, 3, 4\}$

$$\left.\begin{array}{r} 10_2 \\ 11_2 \\ \oplus\ 100_2 \\ \hline 101_2 = 5 \end{array}\right\}$$ Nimber of this position is $*5$

Winning move consists of changing 100_2 to $(100_2 \oplus 101_2) = 01_2$

ie take 3 berries from 3^{rd} heap, leaving 1.

* Note : As far as nimbers go, the game positions $\{2,3,4\}$ and $\{5\}$ have the same nimber, $*5$.

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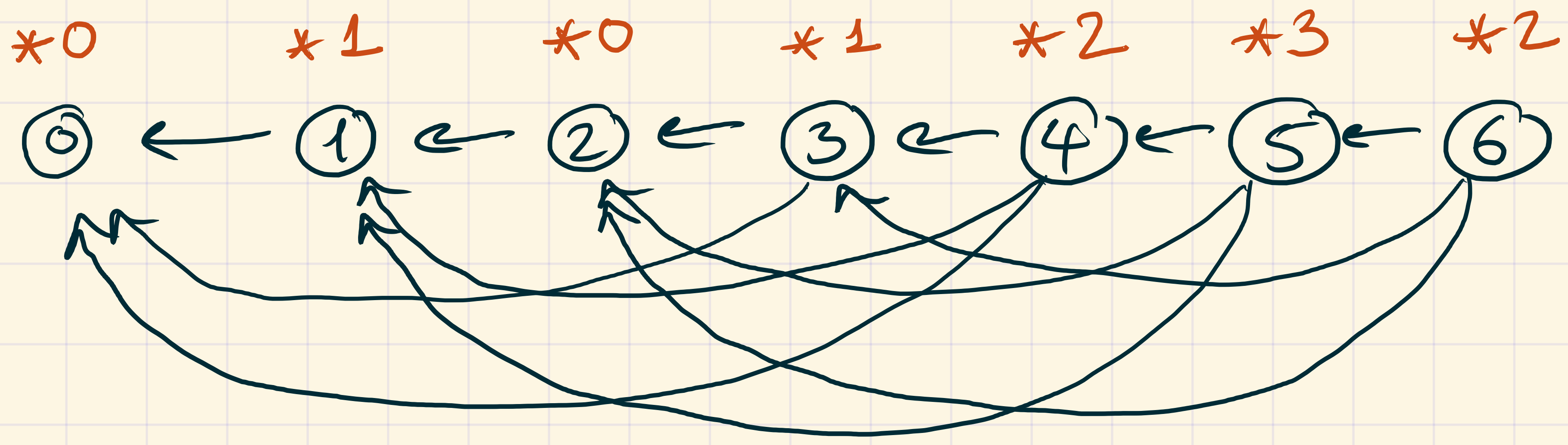
* Grundy labellings (for impartial combinatorial games)

E.g.

Subtraction game $n = 6$

$S = \{1, 3, 4\}$



- Label ⓪ → $*0$
- Label ① → mex $\{*0\}$ $= *1$
- Label ② → mex $\{*1\}$ $= *0$
- Label ③ → mex $\{*0\}$ $= *1$
- Label ④ → mex $\{*0, *1\}$ $= *2$

For Grundy labelling:

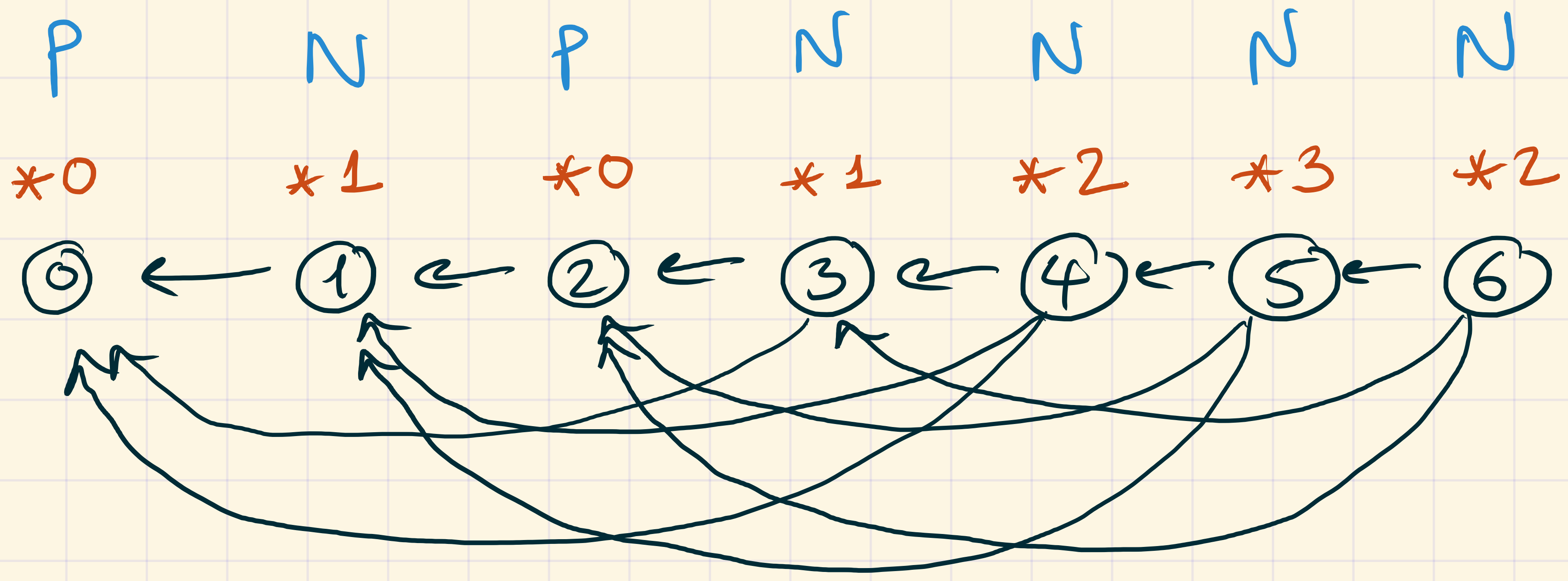
1) Every terminal position (where there are no possible moves) is labelled $*0$

2) Consider a position that can go to positions labelled $*x_1, *x_2, *x_3, \dots *x_k$

Label the current position by the nimber that is the mex of $*x_1, *x_2, \dots *x_k$

mex $\{*x_1, *x_2, \dots, *x_k\}$ = min non-negative nimber that is not in this set.

minimum excluded



Prop: Every position that has a Grundy label of $*0$ is a P position.

Every other position is an N position.

Pf: Note that terminal positions, which are P positions, have a Grundy label of $*0$.

Given any other position with label $*0$.

we know that $*0$ = mex of all outgoing positions,

i.e. all outgoing positions have positive Grundy labels! $\Rightarrow$ $*0$ are P

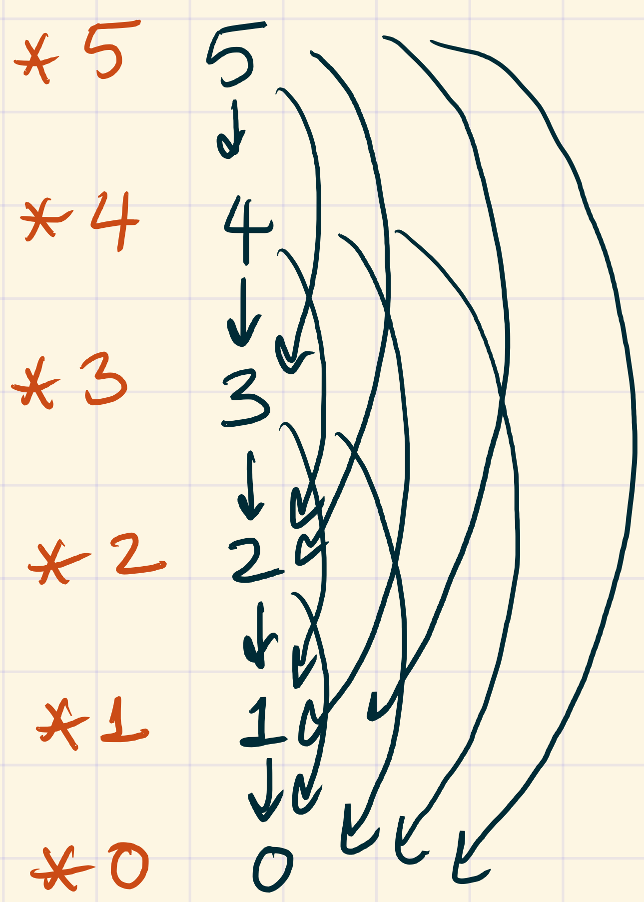
Given any position with label $*k$ for $k \neq 0$,

$*k$ = mex of all outgoing positions

$\Rightarrow$ at least one outgoing position has label $*0$!

$\Rightarrow$ $*k$ are N for $k \neq 0$.

E.g. Nim with 1 pile of size 5



Grundy label is just the pile size!

E.g. Nim with $(1, 2)$