MATH 2301 : Games, graphs, and machines

* Course admin
  - Discussion forum on Zulip ← sign up from Wattle
  - Office hour (time TBA)
  - Notes at https://asilata.github.io/ggm/2021
  - Workshops start in Week 2

* Assessment
  - 40% final exam
  - 25% mid-sem exam
  - 30% assignments
  - 5% workshop participation.

* Outline
  - Basic mathematical language ← sets, relations
  - Posets ← partially ordered sets
  - Graphs
  - Finite automata & regular languages ← machines
  - Game theory ~ combinatorial games

* Why take this course?

  Learn abstraction!

  * Forget the unimportant things and successfully model the important ones.
  * Techniques to model various situations mathematically

* Sets

  Informally: an unordered collection of distinct objects

  Examples:

  $\{1,2,5\}$, $\{\text{Sydney}\}$, $\{x \in \mathbb{N} \mid x \text{ is even}\}$ "set builder notation"

  $\in$: "in"; $\mathbb{N}$: natural numbers

  * Two sets are equal if and only if they have the same elements.

  [Formal axiomatic construction of sets: Zermelo-Fraenkel set theory]

** Set constructions

- Empty set : ϕ = the unique set that has no elements in it.
- Subset : $A \subseteq B$ (or $A \subset B$) if every $x \in A$ is in B
- Superset : $A \supseteq B$ (or $A \supset B$) if every $x \in B$ is in A
- Union : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
  "x such that x is in A or x is in B"
- Intersection : $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Power set of a set A : $P(A) = \{S \mid S \subseteq A\}$
  sets can have other sets as elements!
- Cartesian product of two sets A & B :
  $A \times B = \{(a, b) \mid a \in A, b \in B\}$

** Examples

- $P(\{1\}) = \{\phi, \{1\}\}$
- $\{1,2\} \times \{2,3,4\} = \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4)\}$
- $P(\phi) = \{\phi\}$

  The empty set is a subset of every set!
  $\phi \neq \{\phi\}$
- $\phi \times \{5,6\} = \phi$

  has no elements,
  so there are no possible ordered pairs (a,b) in this product!

* Relations

** Informal meaning : A property / a way that links two or more things together

Examples :

- Two fruits in a single fruit basket are related
- Canberra is related to the ACT because it is in the ACT
- 2 and 2020 are related : they're both even.

** Formal definition?

Defn : A relation R on two sets S & T is simply a subset $R \subseteq (S \times T)$

(More precisely, this a binary relation.)

- If $(a, b) \in R$, we say that aRb (sometimes)
- A binary relation on a set S is just a subset of $S \times S$

** Functions

Suppose $R \subseteq S \times T$. We say that R is a (partial) function if whenever $(a, b) \in R$ and $(a, c) \in R$ for some $b, c \in T$, then we have $b = c$.