# MATH 2301 : Games, graphs, and machines

## * Course admin

- Discussion forum on Zulip ← sign up from Wattle
- Office hour (time TBA)
- Notes at https://asilata.github.io/ggm/2021
- Workshops start in Week 2

## * Assessment

- 40% final exam
- 25% mid-sem exam
- 30% assignments
- 5% workshop participation.

## * Outline

- Basic mathematical language ← sets, relations
- Posets ← partially ordered sets
- Graphs
- Finite automata & regular languages ← machines
- Game theory ← combinatorial games

* **Why take this course?**

Learn abstraction!

* Forget the unimportant things and successfully model the important ones.

* Techniques to model various situations mathematically

* **Sets**

Informally: an unordered collection of distinct objects

Examples: "set builder notation"

$\{1, 2, 5\}$, $\{\text{Sydney}\}$, $\{x \in \mathbb{N} \mid x \text{ is even}\}$

"in" natural numbers

* Two sets are equal if and only if they have the same elements.

[Formal axiomatic construction of sets: Zermelo-Fraenkel set theory]

## ** Set constructions

- Empty set : $\phi$ = the unique set that has no elements in it.

- Subset : $A \subseteq B$ (or $A \subset B$) if every $x \in A$ is in $B$

- Superset : $A \supseteq B$ (or $A \supset B$) if every $x \in B$ is in $A$

- Union : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

  "x such that x is in A or x is in B"

- Intersection : $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

- Power set of a set A : $P(A) = \{S \mid S \subseteq A\}$

  sets can have other sets as elements!

- Cartesian product of two sets A & B :

  $A \times B = \{(a, b) \mid a \in A,\ b \in B\}$

## ** Examples

- $P(\{1\}) = \{\phi, \{1\}\}$

- $\{1,2\} \times \{2,3,4\} = \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4)\}$

- $P(\phi) = \{\phi\}$

  The empty set is a subset of every set!

  $\phi \neq \{\phi\}$

- $\phi \times \{5,6\} = \phi$

  has no elements,

  so there are no possible ordered pairs (a,b) in this product!

# * Relations

## ** Informal meaning : A property / a way that links two or more things together

Examples :

- Two fruits in a single fruit basket are related
- Canberra is related to the ACT because it is in the ACT
- 2 and 2020 are related ; they've both even.

## ** Formal definition?

Defn : A relation $R$ on two sets $S$ & $T$ is simply a subset $R \subseteq (S \times T)$

(More precisely, this a binary relation.)

- If $(a,b) \in R$, we say that $aRb$ (sometimes)
- A binary relation on a set $S$ is just a subset of $S \times S$

## ** Functions

Suppose $R \subseteq S \times T$. We say that $R$ is a (partial) function if whenever $(a,b) \in R$ and $(a,c) \in R$ for some $b,c \in T$, then we have $b = c$.