

Math 2301

* From last time: functions

A function is a relation $R \subseteq S \times T$

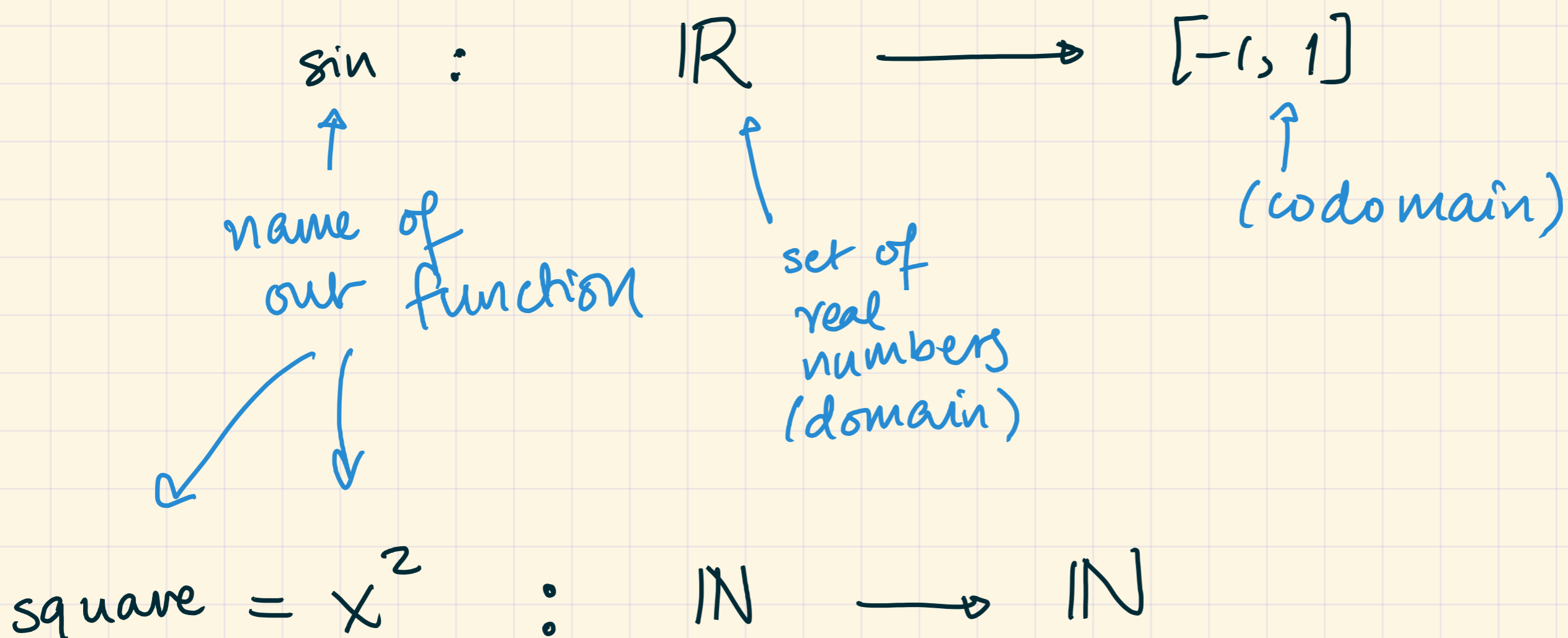
satisfies two additional properties:

(1) If $(s, t_1) \in R$ and $(s, t_2) \in R$ for some $s \in S$ and $t_1, t_2 \in T$, then $t_1 = t_2$
(unique "output" for each "input" that appears)

(2) [Conventionally] for every $s \in S$, there is a $t \in T$ such that $(s, t) \in R$.
(for any "input" s , there is an "output" t)

In this case, we say that S is the domain & T is the codomain, and usually write functions as $f: S \rightarrow T$.

** Example



* Graphs

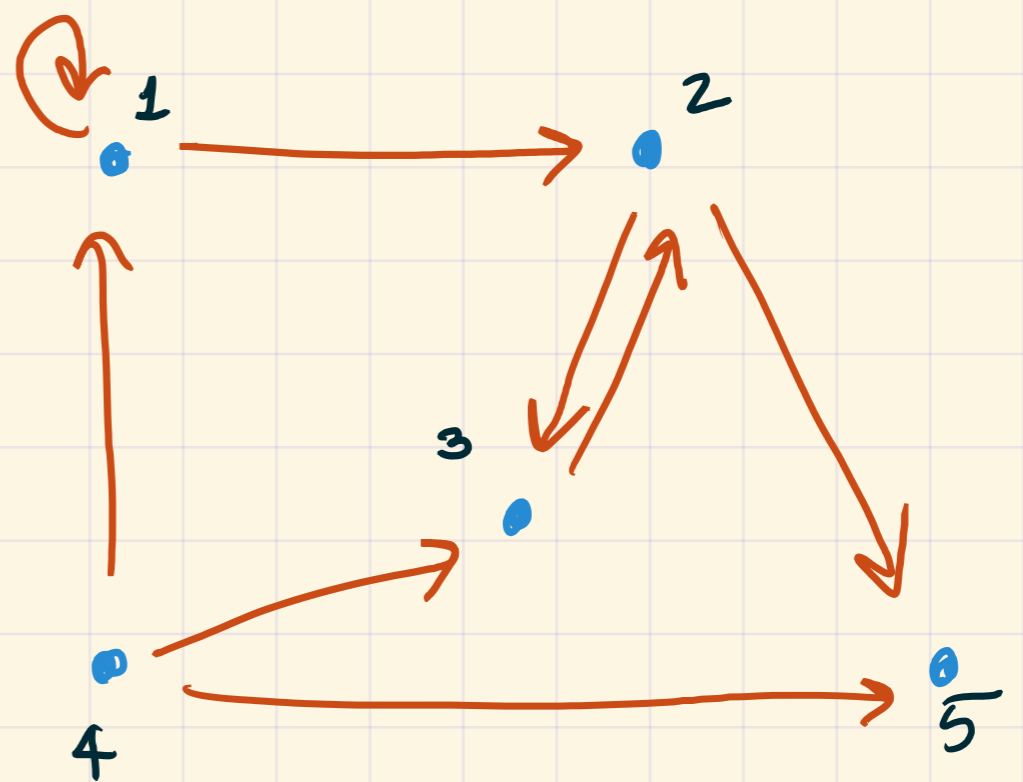
- Used to organise information visually
- Also an excellent computational tool.

** Definition: A directed graph consists of a set V of "vertices"/"nodes", and a binary relation on V , called $E \subseteq V \times V$. The elements of E are ordered pairs of vertices, called "edges".

** Drawing directed graphs:

$$V = \{1, 2, 3, 4, 5\}$$

$$E = \{(1,1), (1,2), (2,3), (2,5), (3,2), (4,1), (4,3), (4,5)\}$$



* Warning: Renumbering the vertices in a drawing, you'll get a graph that looks the same, but has a different V and a different E .

Technically, you'll get a different graph, but it's isomorphic to the first one.

More about this later.

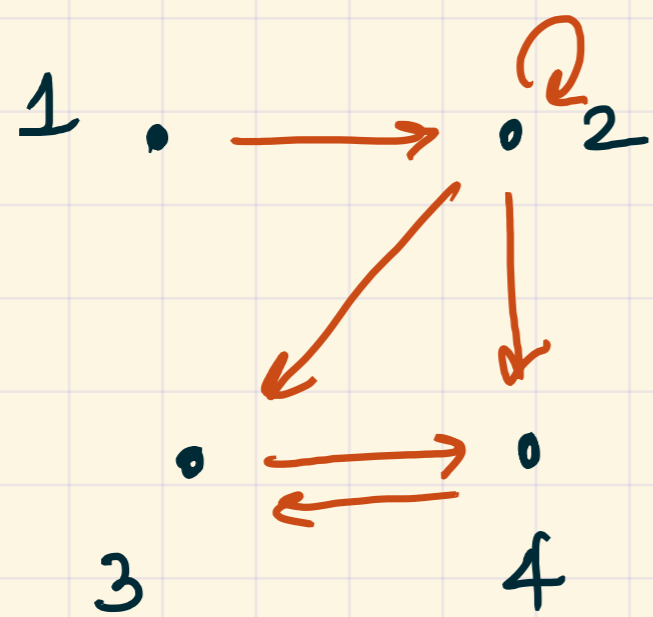
no Many (V,E) pairs give you the same drawing

- At the moment, our definition says there'll be at most one edge in a given direction between two vertices.

** The adjacency matrix

Recall: A matrix is a rectangular array of numbers (typically).

Consider a graph $G = (V, E)$. Choose an



ordering on the set V.

Let n be the number of elements in V .

Consider an $(n \times n)$ matrix

rows \uparrow columns

end

	1	2	3	4
1	0	1	0	0
2	0	1	1	1
3	0	0	0	1
4	0	0	1	0

start

records edges from 1 to other vertices

Defn: Let (V, E) be a graph with n vertices (ordered).

The adjacency matrix is an $n \times n$ matrix whose

$(i, j)^{\text{th}}$ entry is the number of edges from i to j .

* Properties of relations (vis-a-vis the adjacency matrix)

** Fix a set S , and the relation R will be on S .

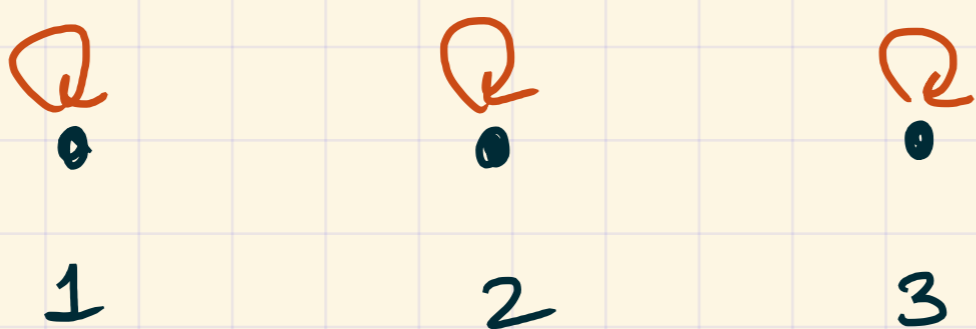
** Reflexivity: A relation is called reflexive if for each $s \in S$, the pair $(s, s) \in R$

*** Example

$$S = \{1, 2, 3\}, \quad R = \{(1, 1), (2, 2), (3, 3)\}$$

"x=y"

*** Graph



Q [workshop]

What property should the adjacency matrix satisfy?

** Symmetry

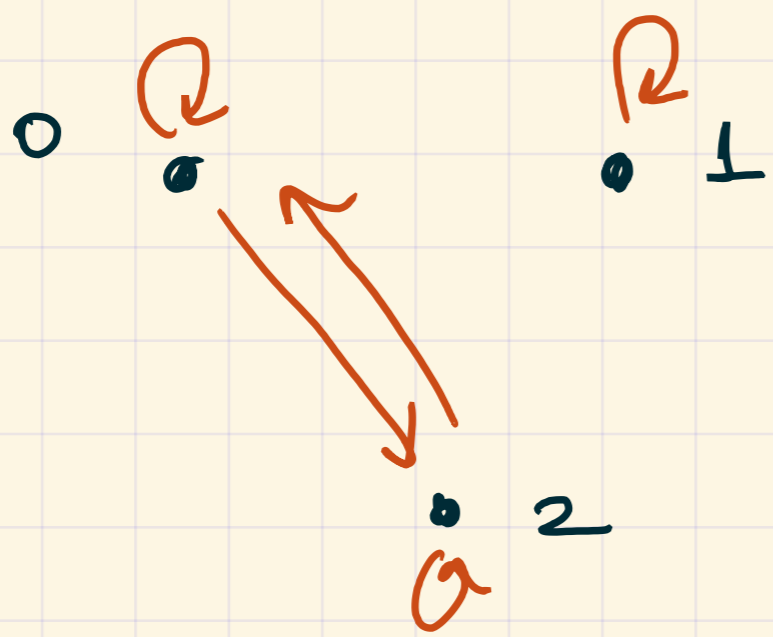
Defn: R is symmetric if whenever $(x, y) \in R$, we have $(y, x) \in R$.

*** Example

$$S = \{0, 1, 2\}; \quad (x, y) \in R \text{ if } x+y \text{ is even.}$$

(Equivalently, $y+x$ is even, because $x+y = y+x$.)

*** Graph



Q: [workshop]

Adjacency matrix?

*** Anti-symmetry [bad name "]

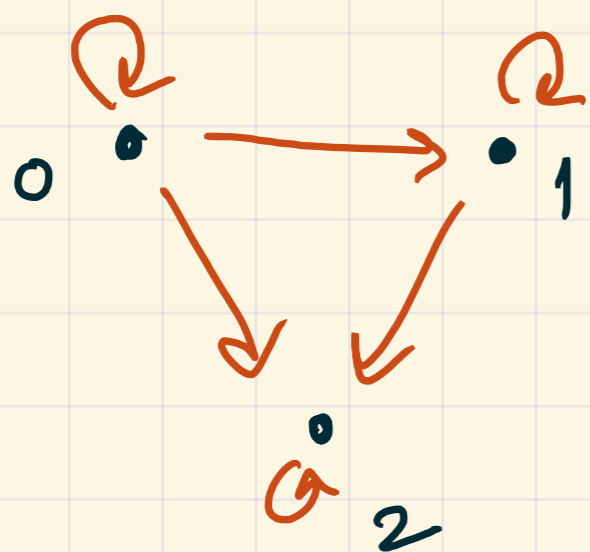
Defn: R is anti-symmetric if whenever $(x, y) \in R$ and $x \neq y$, the pair $(y, x) \notin R$.

Warning: A relation can be both symmetric and anti-symmetric! \rightarrow workshop.

*** Example

$S = \{0, 1, 2\}$, $(x, y) \in R$ if $x \leq y$.

*** Graph



Q [workshop]

Adjacency matrix?

** Being a function

*** Example

$$f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$$

$$f(x) = 4 - x$$

*** Graph

