MATH 2301

* Last time : Intro to posets

* Today : Graphs

** Recall the definition :

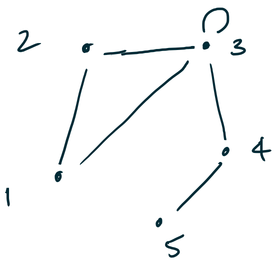
A directed graph consists of V, a vertex set, and $E \subseteq V \times V$, the edge set

Typically, we can draw graphs:

* self-loops ok

* parallel edges not ok

** Undirected graphs



An undirected graph is just a graph $G = (V, E)$, with the restriction that E is a symmetric relation.

$V = \{1, 2, 3, 4, 5\}$

$E = \{(1,2), (2,1), (1,3), (3,1), (2,3), (3,2), (3,3), (3,4), (4,3), (4,5), (5,4)\}$.

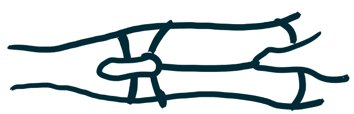
** Questions and applications

- Railway / flight networks
- Water / gas / electricity networks
- Traffic
- Facebook friend graph
- Internet

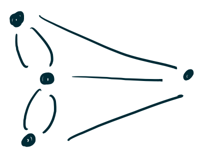
. . .

** Questions

1) Route from A to B?
2) Shortest path?
3) How many paths?
4) How to optimise traffic? (Flow problems?)
5) Can you visit each vertex exactly once? (Hamiltonian path problem)
6) Shortest circuit that visits each vertex? (Travelling salesman problem)
7) Can you visit each edge exactly once? (Eulerian path problem)



Königsberg bridge problem



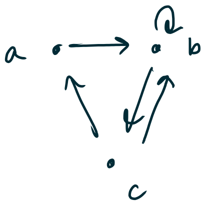
8) Can you connect edges w/o overlaps?

Planarity?

9) Clusters in a graph?

Are there natural groupings in the graph?

** Adjacency matrix.



$$
\begin{array}{c|ccc}
 & a & b & c \\
\hline
a & 0 & 1 & 0 \\
b & 0 & 1 & 1 \\
c & 1 & 1 & 0
\end{array}
$$

** Matrix sum & product

*** Addition

Let A be an $m \times n$ matrix

number of rows

number of columns

Let B be another $m \times n$ matrix.

Then we can add:

$$(A+B)_{(i,j)} := A_{(i,j)} + B_{(i,j)}$$

defining the entry in the i^{th} row & j^{th} column of $(A+B)$

Example $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 5 \\ 4 & -3 & 2 \end{bmatrix}$

$$(A+B) = \begin{bmatrix} 1 & 3 & 8 \\ 4 & -2 & 1 \end{bmatrix}$$

** Multiplication

Let A be an $m \times \underline{k}$ matrix

Let B be a $\underline{k} \times n$ matrix

Then you can multiply A & B (in that order!)

Example

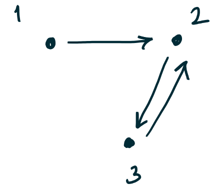
$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ (2×3) $B = \begin{bmatrix} 4 & 0 \\ -1 & 5 \\ 2 & 1 \end{bmatrix}$ (3×2)

$$(A \times B) = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -1 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2\cdot 4 + 1\cdot(-1) + 3\cdot 2 & 2\cdot 0 + 1\cdot 5 + 3\cdot 1 \\ 0\cdot 4 + 1\cdot(-1) + 2\cdot 2 & 0\cdot 0 + 1\cdot 5 + 2\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8-1+6 & 0+5+3 \\ 0-1+4 & 0+5+2 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 3 & 7 \end{bmatrix}$$

Def: If A is $m \times k$ & B is $k \times n$, then $(A \times B)$ is an $m \times n$ matrix.
The $(i,j)^{th}$ entry of $(A \times B)$ is the dot product of the i^{th} row of A with the j^{th} column of B.

** Powers of the adjacency matrix



$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Compute A^2:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q: What do the entries mean?