

MATH 2301

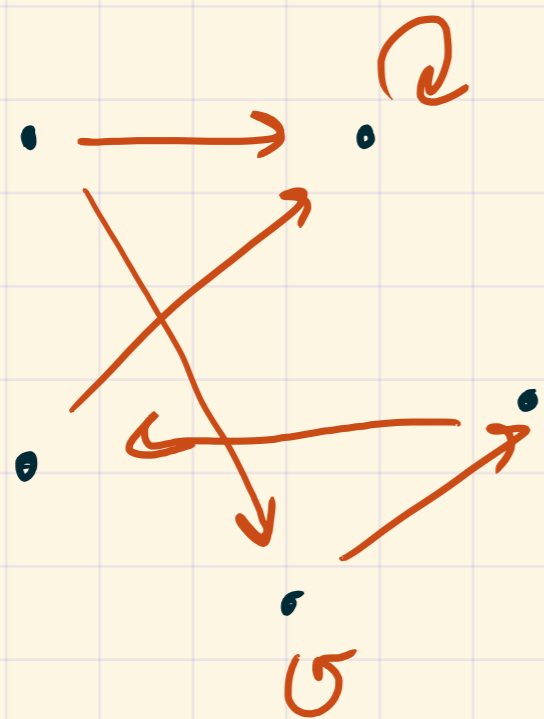
* Last time : Intro to posets

* Today : Graphs

** Recall the definition :

A directed graph consists of V , a vertex set, and $E \subseteq V \times V$, the edge set

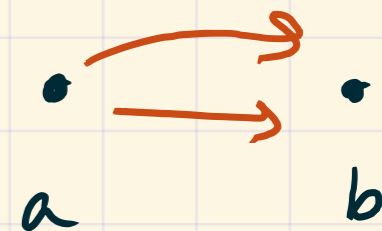
Typically, we can draw graphs:



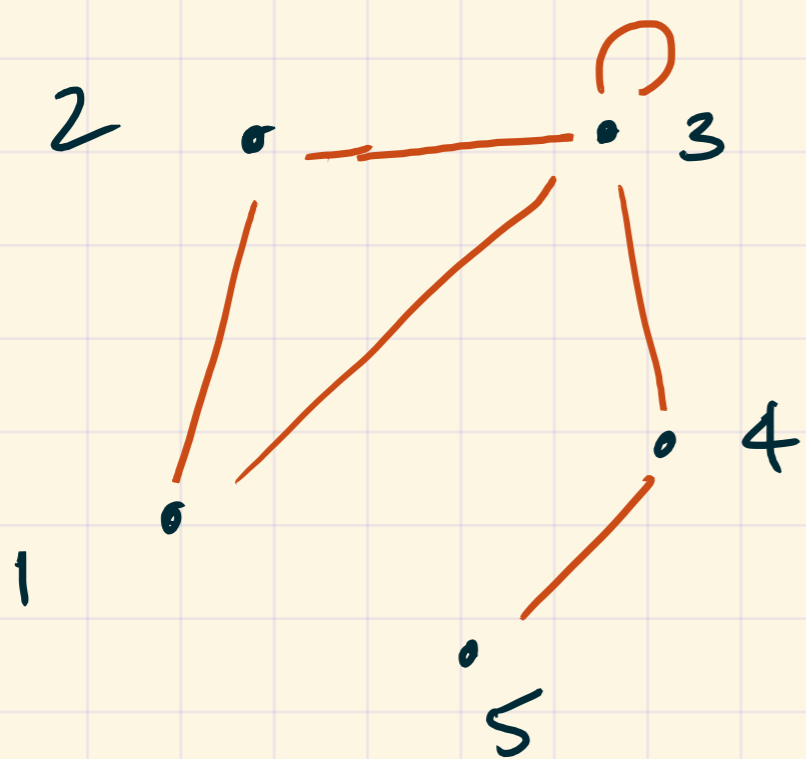
* self-loops OK



* parallel edges NOT OK



** Undirected graphs



An undirected graph is just a graph $G = (V, E)$, with the restriction that E is a symmetric relation.

$$V = \{1, 2, 3, 4, 5\}$$

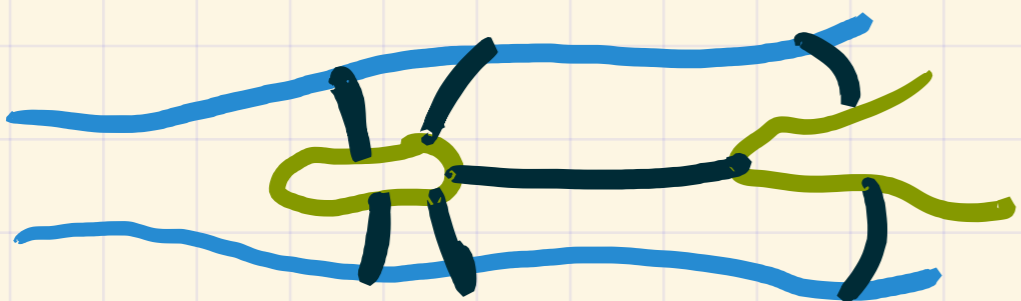
$$E = \{ (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (3, 3), (3, 4), (4, 3), (4, 5), (5, 4) \}.$$

** Questions and applications

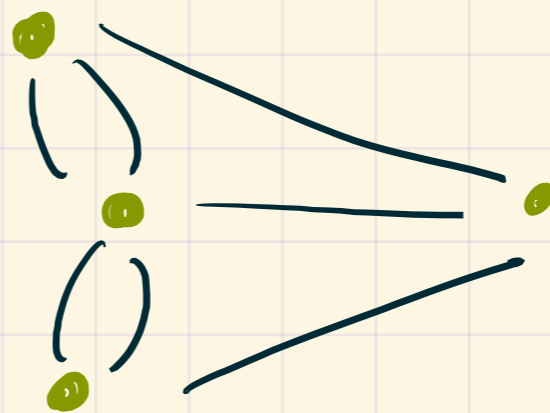
- Railway / flight networks
- Water / gas / electricity networks
- Traffic
- Facebook friend graph
- Internet
- ...

** Questions

- 1) Route from A to B?
- 2) Shortest path?
- 3) How many paths?
- 4) How to optimise traffic? (Flow problems?)
- 5) Can you visit each vertex exactly once?
(Hamiltonian path problem)
- 6) Shortest circuit that visits each vertex?
(Travelling salesman problem)
- 7) Can you visit each edge exactly once?
(Eulerian path problem)



Königsberg bridge problem



8) Can you connect edges w/o overlaps?

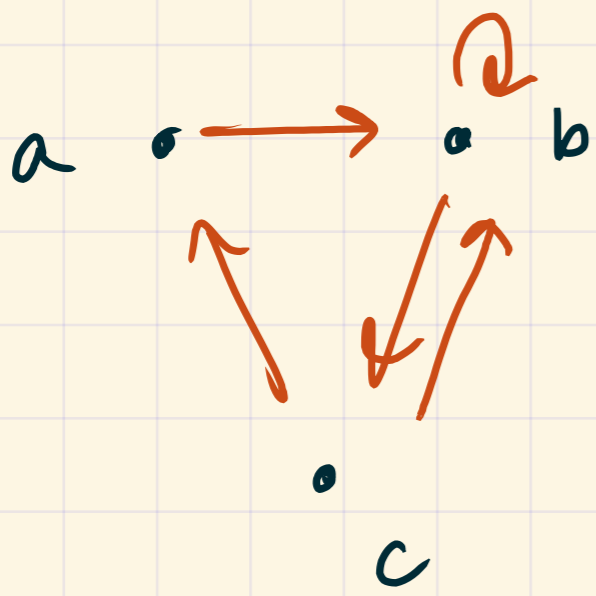


Planarity?

9) Clusters in a graph?

Are there natural groupings in the graph?

** Adjacency matrix.



$$\begin{matrix} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

** Matrix sum & product

*** Addition Let A be an $m \times n$ matrix
number of rows \uparrow number of columns \uparrow

Let B be another $m \times n$ matrix.

Then we can add:

$$(A+B)_{(i,j)} := A_{(i,j)} + B_{(i,j)}$$

\uparrow
defining the entry
in the i^{th} row &
 j^{th} column of $(A+B)$

Example $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 5 \\ 4 & -3 & 2 \end{bmatrix}$

$$(A+B) = \begin{bmatrix} 1 & 3 & 8 \\ 4 & -2 & 1 \end{bmatrix}$$

** Multiplication

Let A be an $m \times \underline{k}$ matrix

Let B be a $\underline{k} \times n$ matrix

Then you can multiply A & B (in that order!)

Example

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix} \quad (2 \times 3)$$

$$B = \begin{bmatrix} 4 & 0 \\ -1 & 5 \\ 2 & 1 \end{bmatrix} \quad (3 \times 2)$$

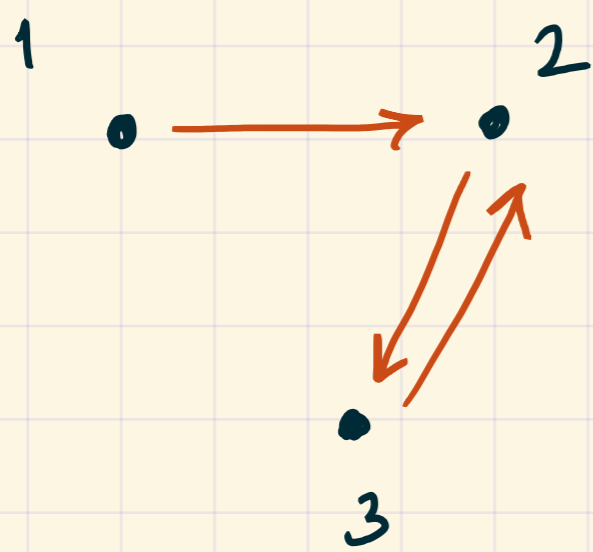
$$(A \times B) = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -1 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 \cdot 4 & 2 \cdot 0 \\ + 1 \cdot (-1) & + 1 \cdot 5 \\ + 3 \cdot 2 & + 3 \cdot 1 \\ 0 \cdot 4 & 0 \cdot 0 \\ + 1 \cdot (-1) & + 1 \cdot 5 \\ + 2 \cdot 2 & + 2 \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 - 1 + 6 & 0 + 5 + 3 \\ 0 - 1 + 4 & 0 + 5 + 2 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 3 & 7 \end{bmatrix}$$

Def: If A is $m \times k$ & B is $k \times n$, then $(A \times B)$ is an $m \times n$ matrix.

The $(i, j)^{\text{th}}$ entry of $(A \times B)$ is the dot product of the i^{th} row of A with the j^{th} column of B .

** Powers of the adjacency matrix



$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Compute A^2 :

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q: What do the entries mean?