

MATH 2301

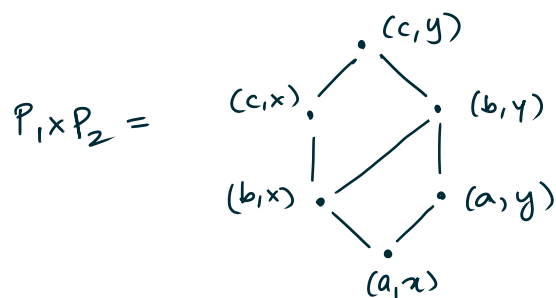
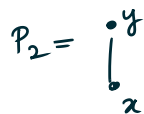
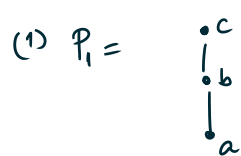
* Exam this evening @ 6:30 pm. (unless otherwise arranged)

- Zoom details on Wattle
- Keep video ON and mic MUTED.
- Communicate with invigilators via zoom chat only.

* Continued: products of posets

** Def: Let (P_1, \leq_1) & (P_2, \leq_2) be posets. The product poset is defined as the set $P_1 \times P_2$ with the relation $(a, b) \leq (c, d)$ if $a \leq_1 c$ and $b \leq_2 d$.

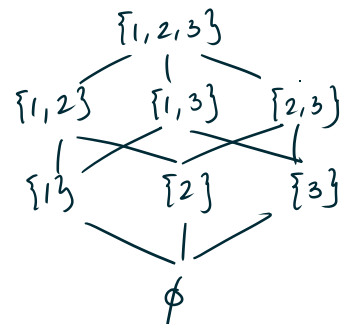
** Example



(2) The subset poset

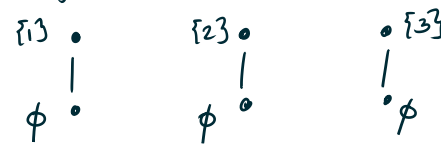
Let $P = \{ \}$

$A = \{1, 2, 3\}$

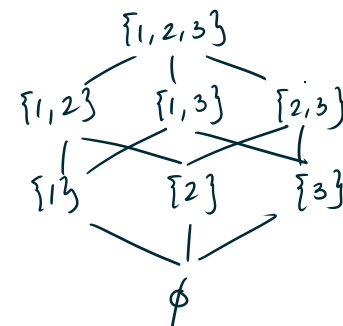
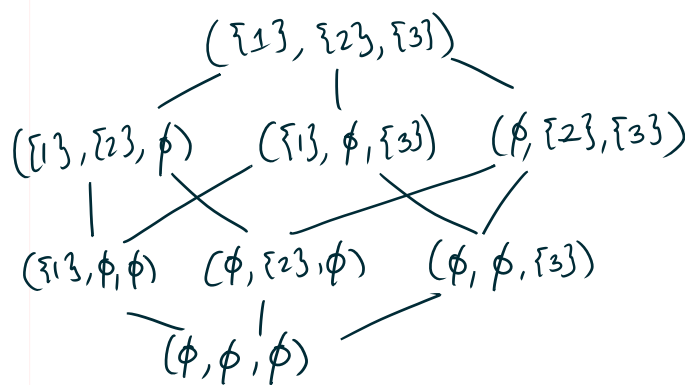


Claim: The subset poset of A has the same Hasse diagram as $(P \times P \times P)$

Label the 3 copies of P as follows:



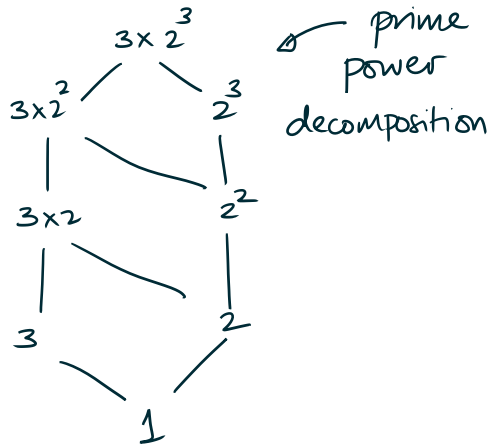
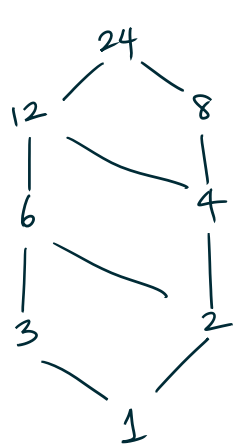
Let's draw $P \times P \times P$. Compare w/ previous.



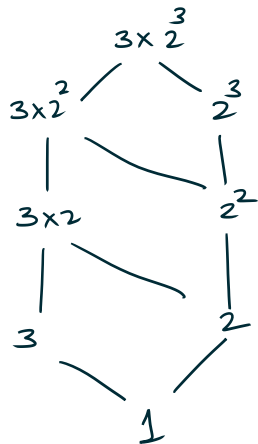
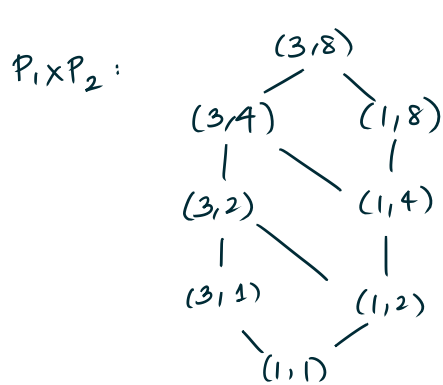
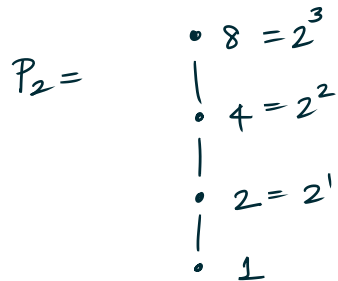
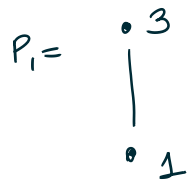
Left hand side \rightarrow right hand side is by taking set union of all coordinates.

** Theorem: Let A be a set of n elements. Then the subset poset of A has the same Hasse diagram (same shape) as the product $(P \times P \times \dots \times P)$ $\underbrace{\hspace{2cm}}_{n \text{ times}}$.

(5) The divisor poset



prime power decomposition



LHS \rightarrow RHS by taking product of the coordinates.

** Theorem : Let $m \geq 1$ be an integer. Suppose

$$m = p_1^{a_1} \cdot p_2^{a_2} \cdots p_k^{a_k}$$

be its prime power decomposition: p_1, \dots, p_k all prime, and $p_1 < p_2 < \dots < p_k$

Let P_1, P_2, \dots, P_k be the divisor posets of $p_1^{a_1}, p_2^{a_2}, \dots, p_k^{a_k}$.

Then the shape of the Hasse diagram for the divisor poset of m is the same as that for $P_1 \times P_2 \times \dots \times P_k$.

** Observe : The divisor poset of any prime power p^n is a linear (total order):



** Let's go back to computing μ . (We'll use the previous observation)

Recall : μ is the inverse of ζ .

** Theorem: Let (P_1, \leq) and (P_2, \leq) be posets.

Let μ_1 and μ_2 be the μ functions of P_1 & P_2 respectively.

Let μ be the μ -function for $P_1 \times P_2$. Then, for any interval $[(a,b), (c,d)]$ in $P_1 \times P_2$, we have:

$$\mu([(a,b), (c,d)]) = \mu([a,c]) \cdot \mu_2([b,d])$$

** Corollary

(1) Let A be a set w/ n elements. Let $X \subseteq Y$ be subsets of A .

$$\boxed{\mu([X, Y]) = (-1)^{|Y \setminus X|}}$$

E.g. $X = \{3\}$ & $Y = \{1, 2, 3\}$

$$\downarrow$$

$$(\emptyset, \emptyset, \{3\})$$

$$\downarrow$$

$$(\{1\}, \{2\}, \{3\})$$

$$\begin{aligned} \mu([X, Y]) &= \underbrace{\mu([\emptyset, \{1\}])}_{=-1} \cdot \underbrace{\mu([\emptyset, \{2\}])}_{=-1} \cdot \underbrace{\mu([\{3\}, \{3\}])}_{=1} \\ &= 1 \end{aligned}$$

(2) Let m, n be positive integers.

$$n = p_1^{a_1} \cdots p_k^{a_k}, \quad m = p_1^{b_1} p_2^{b_2} \cdots p_k^{b_k}$$

$$\mu([1, n]) = \mu([1, p_1^{a_1}]) \cdots \mu([1, p_k^{a_k}])$$

$$\mu([n, m]) = \mu([p_1^{a_1}, p_1^{b_1}]) \cdots \mu([p_k^{a_k}, p_k^{b_k}])$$

(Let's finish this on Wed.)