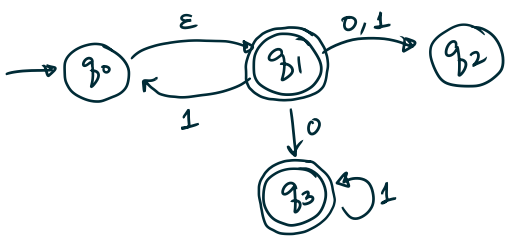
MATH 2301

* Non-deterministic finite automata (NFA).

** Informal description



It is almost exactly like a DFA, with the following key differences:

1) Arrows may be labelled by ε
2) Each state can have zero or more outgoing arrows labelled by each letter of Σ.

** Note: An NFA has only one start state, and any number of accept states.

** How to compute with an NFA?

Given any string w, try all possible ways to run w through the NFA.
If at least one option reaches an accept state, and you have read all of w, then ACCEPT w.
Otherwise, REJECT w.

** Formal definition

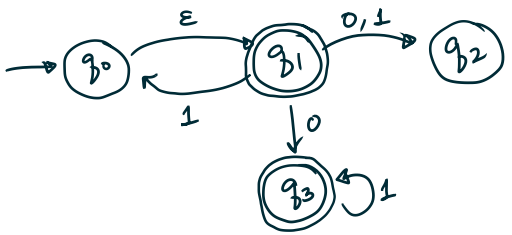
Fix an alphabet Σ

*** Def: A nondeterministic finite automaton (NFA) consists of the following:

1) A set of states Q.
2) A single start state $q_0 \in Q$
3) A set of accept states $A \subseteq Q$.
4) A transition function

$$\Delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$$

- Q: state you're at
- $\Sigma \cup \{\varepsilon\}$: the letter (or ε) that you read
- $\mathcal{P}(Q)$: power set of Q.



Example values of Δ

$\Delta(q_0, 1) = \phi \in \mathcal{P}(Q)$

$\Delta(q_0, 0) = \phi$

$\Delta(q_0, \varepsilon) = \{q_1\}$

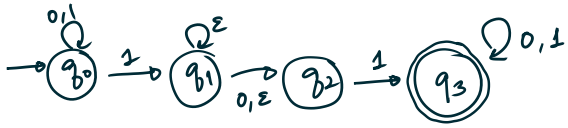
$\Delta(q_1, 1) = \{q_0, q_2\}$

$\Delta(q_1, 0) = \{q_2, q_3\}$

$\Delta(q_3, 1) = \{q_3\}$

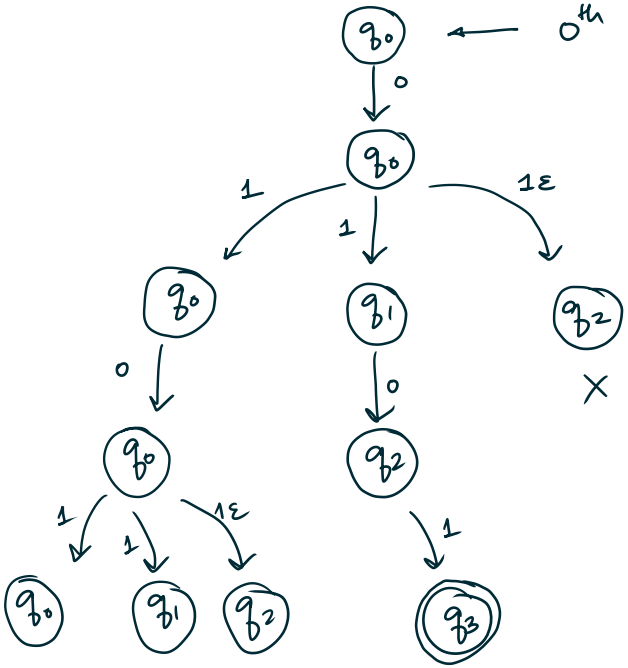
$\Delta(q_2, \varepsilon) = \phi$.

** Example



$w = 0101$

Draw a calculation tree:



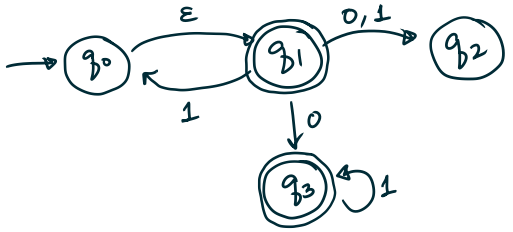
Result: ACCEPT, because q_3 is reachable at level 4, and is accepting.

** Key points about the calculation / calculation tree

- When you read a letter, you should also read any εs that come before & after.

E.g. If you're reading "1", you should follow all paths of the form $\underbrace{\varepsilon\varepsilon\cdots\varepsilon}_{\text{any number of } \varepsilon}\,1\,\underbrace{\varepsilon\cdots\varepsilon\varepsilon}_{\text{any number of } \varepsilon}$

- self-loops labelled ε can be ignored.
- Branches that don't reach the last level (i.e. the end of the string) are failed branches.
- If none of the branches reach the last level, reject.



$w = 110$