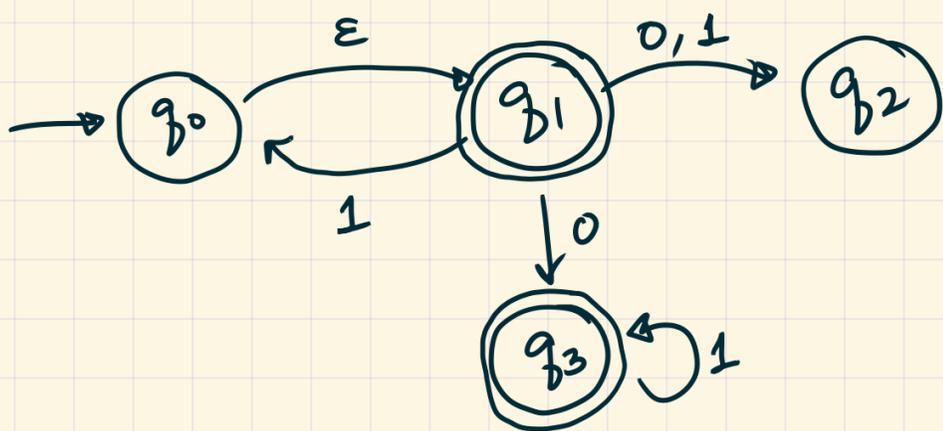


MATH 2301

* Non-deterministic finite automata. (NFA).

** Informal description



It is almost exactly like a DFA, with the following key differences:

- 1) Arrows may be labelled by ϵ
- 2) Each state can have zero or more outgoing arrows labelled by each letter of Σ .

** Note: An NFA has only one start state, and any number of accept states.

** How to compute with an NFA?

Given any string w , try all possible ways to run w through the NFA.

If at least one option reaches an accept state, and you have read all of w , then ACCEPT w .

Otherwise, REJECT w .

** Formal definition

Fix an alphabet Σ

*** Def: A nondeterministic finite automaton (NFA) consists of the following:

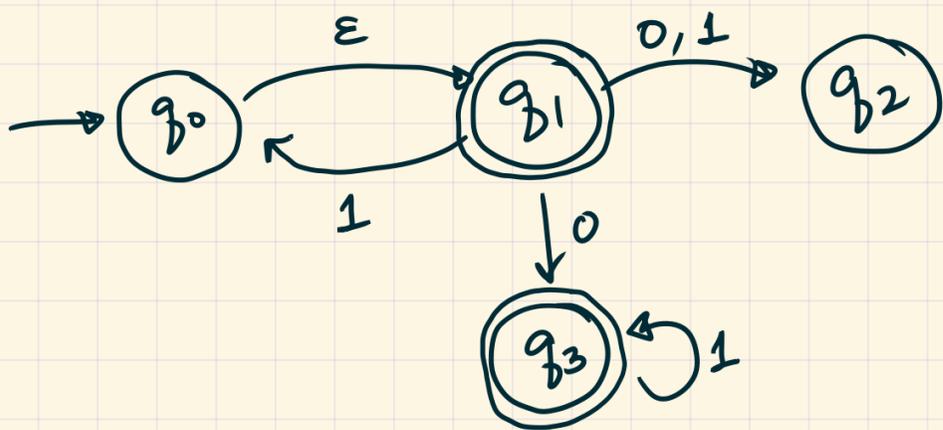
- 1) A set of states Q .
- 2) A single start state $q_0 \in Q$
- 3) A set of accept states $A \subseteq Q$.
- 4) A transition function

$$\Delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow \mathcal{P}(Q)$$

↑
state
you're at

↑
the letter
(or ϵ) that
you read

↑
power set of
 Q .



Example values of Δ

$$\Delta(q_0, 1) = \emptyset \in \mathcal{P}(Q)$$

$$\Delta(q_0, 0) = \emptyset$$

$$\Delta(q_0, \epsilon) = \{q_1\}$$

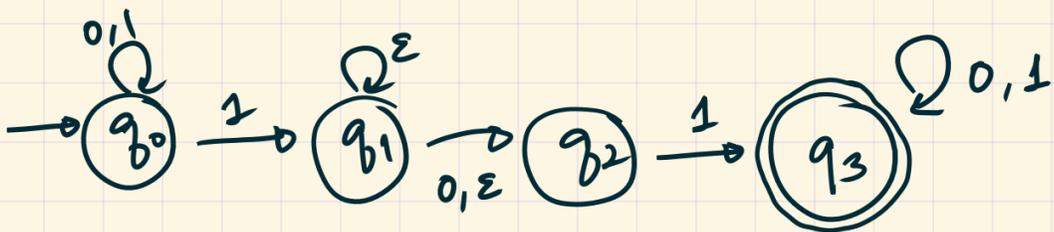
$$\Delta(q_1, 1) = \{q_0, q_2\}$$

$$\Delta(q_1, 0) = \{q_2, q_3\}$$

$$\Delta(q_3, 1) = \{q_3\}$$

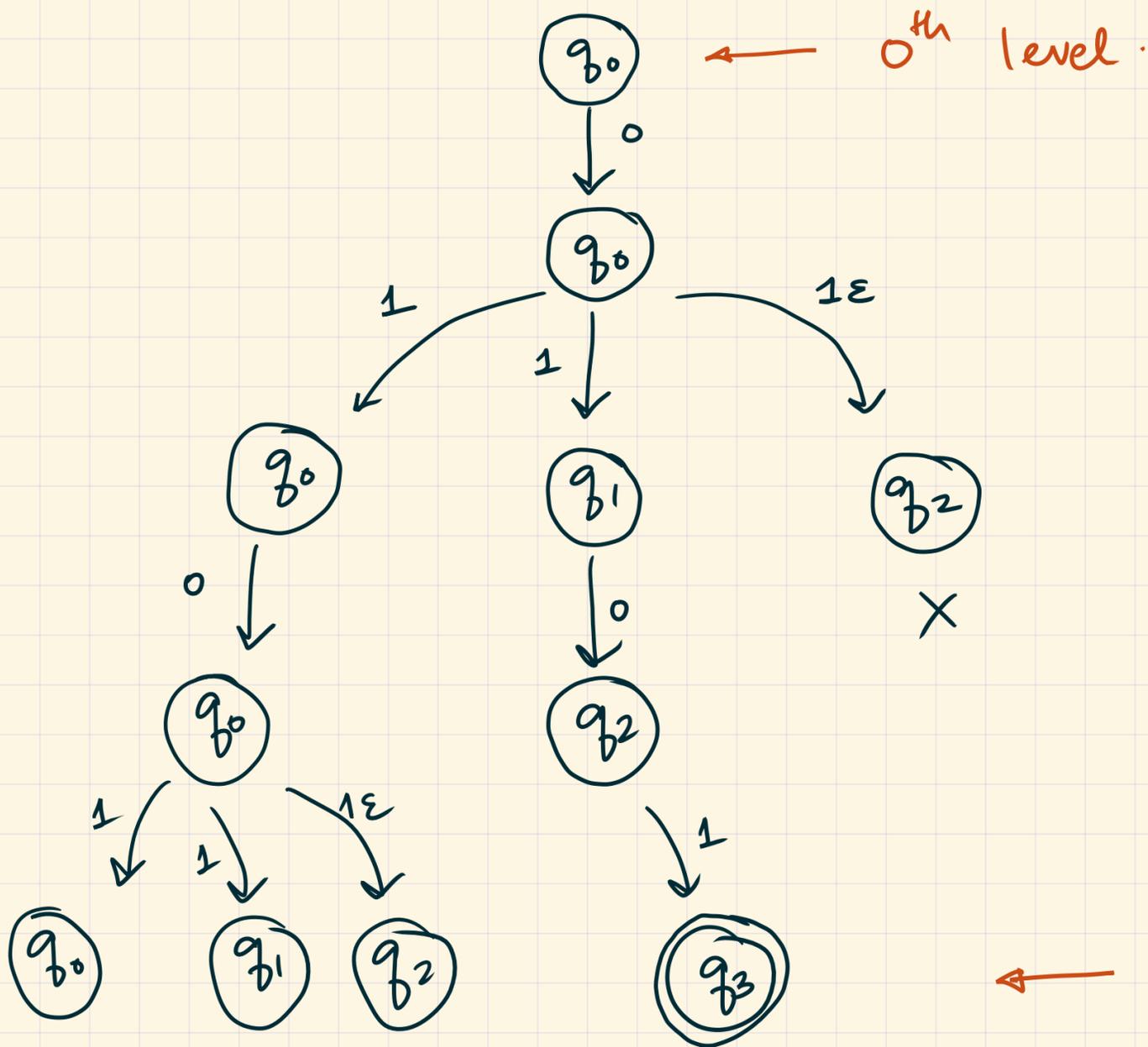
$$\Delta(q_2, \epsilon) = \emptyset$$

** Example



$w = 0101$

Draw a calculation tree :



Result : ACCEPT, because q_3 is reachable at level 4, and is accepting.

End of the calculation (4th level, since $|w| = 4$).

** Key points about the calculation/calculation tree

- When you read a letter, you should also read any ϵ s that come before & after.

E.g. If you're reading "1", you should

follow all paths of the form $\underbrace{\varepsilon\varepsilon\dots\varepsilon}_\text{any number of } \varepsilon 1 \underbrace{\varepsilon\dots\varepsilon}_\text{any number of } \varepsilon$

- self-loops labelled ε can be ignored.
- Branches that don't reach the last level (i.e. the end of the string) are failed branches.
- If none of the branches reach the last level, reject.

