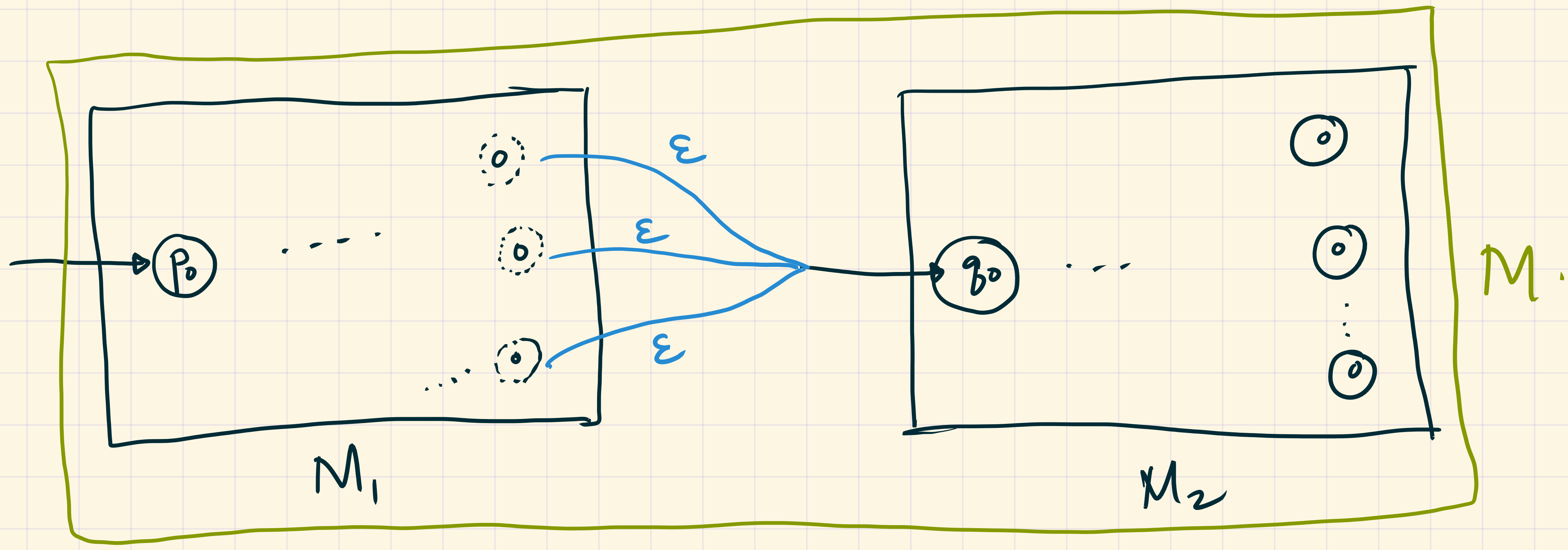
MATH 2301

* Today: Use NFAs to simulate the remaining regexes.

* Back to (4): $r = r_1 r_2$

Suppose we have (DFAs or NFAs) M_1, M_2 with $L(M_1) = L(r_1)$ and $L(M_2) = L(r_2)$. Want M, such that $L(M) = L(M_1) \circ L(M_2)$.



Connect every accept state of M_1 to the start state of M_2 by an arrow labelled ε.

Then change every accepting state of M_1 to a rejecting state.

** Rmk: If M_1 has no accept states, then M_2 is inaccessible $\Rightarrow L(M) = \phi$

(This is ok: $L(M_1) = \phi$, $L(M_2)$ is something, $L(M_1) \circ L(M_2) = \phi \circ L(M_2) = \phi$.)

** Why does M work?

1) If $w = xy$ where $x \in L(M_1)$, $y \in L(M_2)$, then $w \in L(M)$.

2) If $w \notin L(M_1) \circ L(M_2)$, then $w \notin L(M)$.
Equivalently, if $w \in L(M)$, then w has a splitting $w = xy$ as above.

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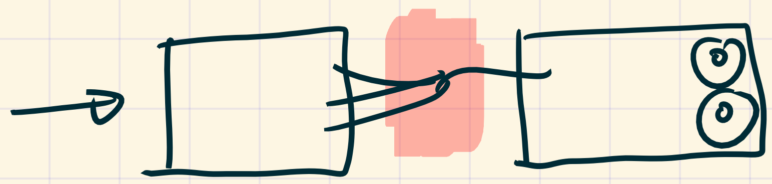
1) Suppose $w = xy$, where $x \in L(M_1)$ & $y \in L(M_2)$.

Then among all possible branches of the calculation tree, at least one does the following:

a) Run x through "M_1", ending at one of the old accepting states of M_1

b) Portal to q_0 by an ε arrow.

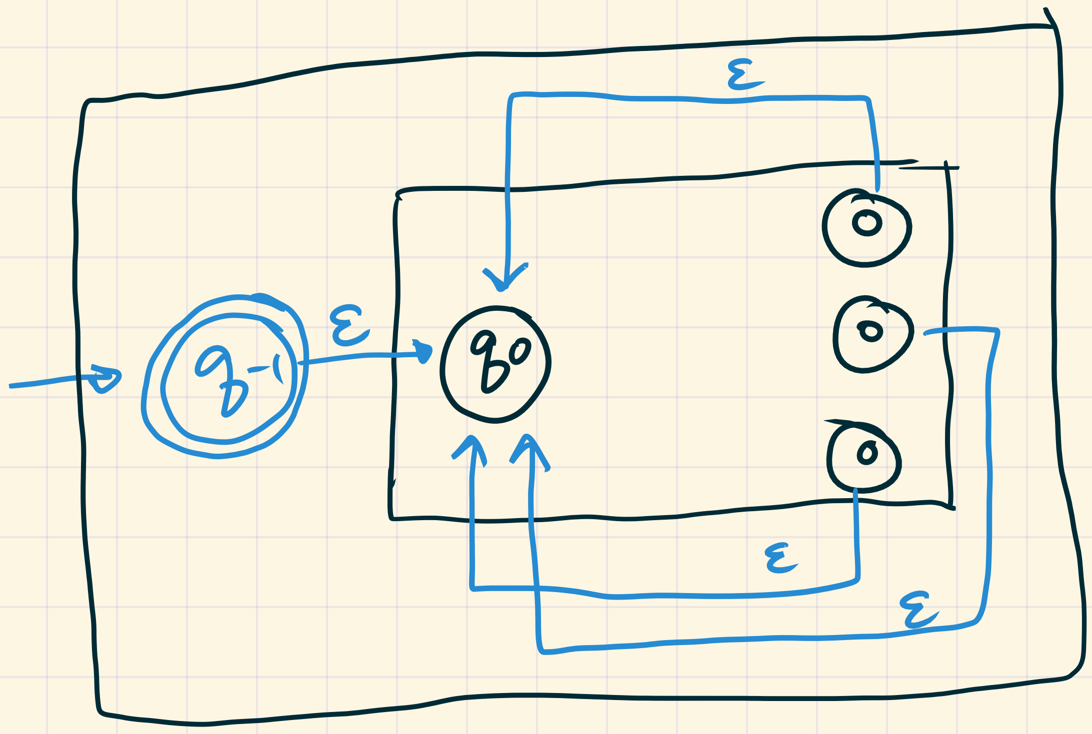
c) Run y through the M_2 portion, ending at an accept state of M_2, hence of M.

2) 

Any accepting branch for w must have a step that passes through the middle portion (highlighted). These are all ε-arrows.
This gives the break point: the portion of w before this is a valid choice for x, and the rest is y.

* Case (6) : $r = r_1^*$.

Given M_1 with $L(M_1) = L(r_1)$, construct M such that $L(M) = L(M_1)^*$.



M

① Connect each accepting state of M_1 to the start state of M_1 by ε arrows.

② Make a new start state q_{-1}, and connect it to q_0 by an ε arrow. [q_0 is no longer the start state.]

③ Make q_{-1} accepting.

** Check [sketch]:

- M accepts ε
- If $|w| > 0$, then M accepts w if and only if $w = x_1 \cdots x_k$, where each $x_i \in L(M_1)$.

(Similar argument to the previous case.)

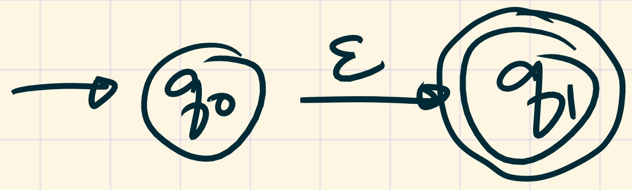
** Summary : We have converted every regex construction into a DFA or NFA construction

⇒ NFAs are at least as powerful as regexes!

* Question: What is the relationship between DFAs & NFAs?

** Easy observation: Every DFA is automatically an NFA; it satisfies the definition automatically.

** Easy observation: There are NFAs that are not DFAs:

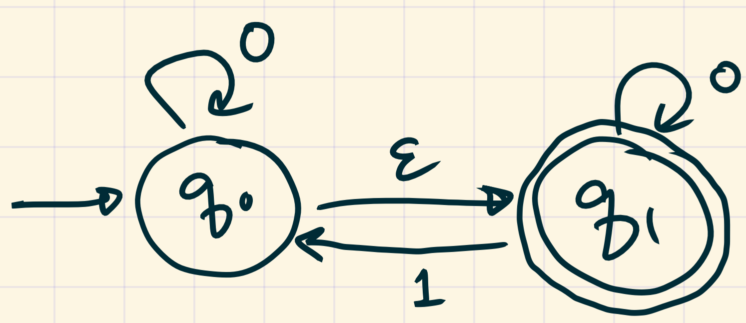


** Theorem: If M is an NFA, we can always find a DFA M' such that

$$L(M') = L(M).$$

In other words, any NFA can always be simulated by a DFA.

** Example



States $Q = \{q_0, q_1\}$

Start state q_0

Set of accept states

$A = \{q_1\}$

Transition function

$\Delta : Q \times (\Sigma \cup \{\varepsilon\}) \to P(Q)$.

E.g. $w = 01$