MATH 2301

* Games

A game is (informally) a model of interactions between two (or more) players, via a prescribed set of moves.

Many kinds of games are studied in mathematics, economics, computer science, finance, etc.

They can be classified on various axes, such as:

- perfect info vs incomplete info
  (all info is known vs unknown)
- simultaneous vs turn-based
- impartial vs partizan ← e.g. chess
  each player either has the same set of rules vs a different set of rules.
- symmetric vs asymmetric
  each player either gets the same payoff or different payoffs for making their moves

** Impartial combinatorial games

We will focus on these. An impartial combinatorial game is played between two players, say A & B

- A & B take turns to modify the "game state".

- The same rules for moves apply to both players.

- The game is perfect-information (no secrets)

- It is deterministic (no chance/luck)

- A player "loses" if there are no valid moves available.

- There are finitely many game states reachable from the starting state

- There can be no backtracking between game states [i.e. no directed cycles of moves]
This ensures that the game always ends.

** Example : Subtraction game.

$S = \{1, 3, 4\}$ ⇝ set of positive naturals

$n = 10$ ⇝ starting value.

Rule for moving: Subtract any one element of S from the current value of n, keeping the result non-negative.

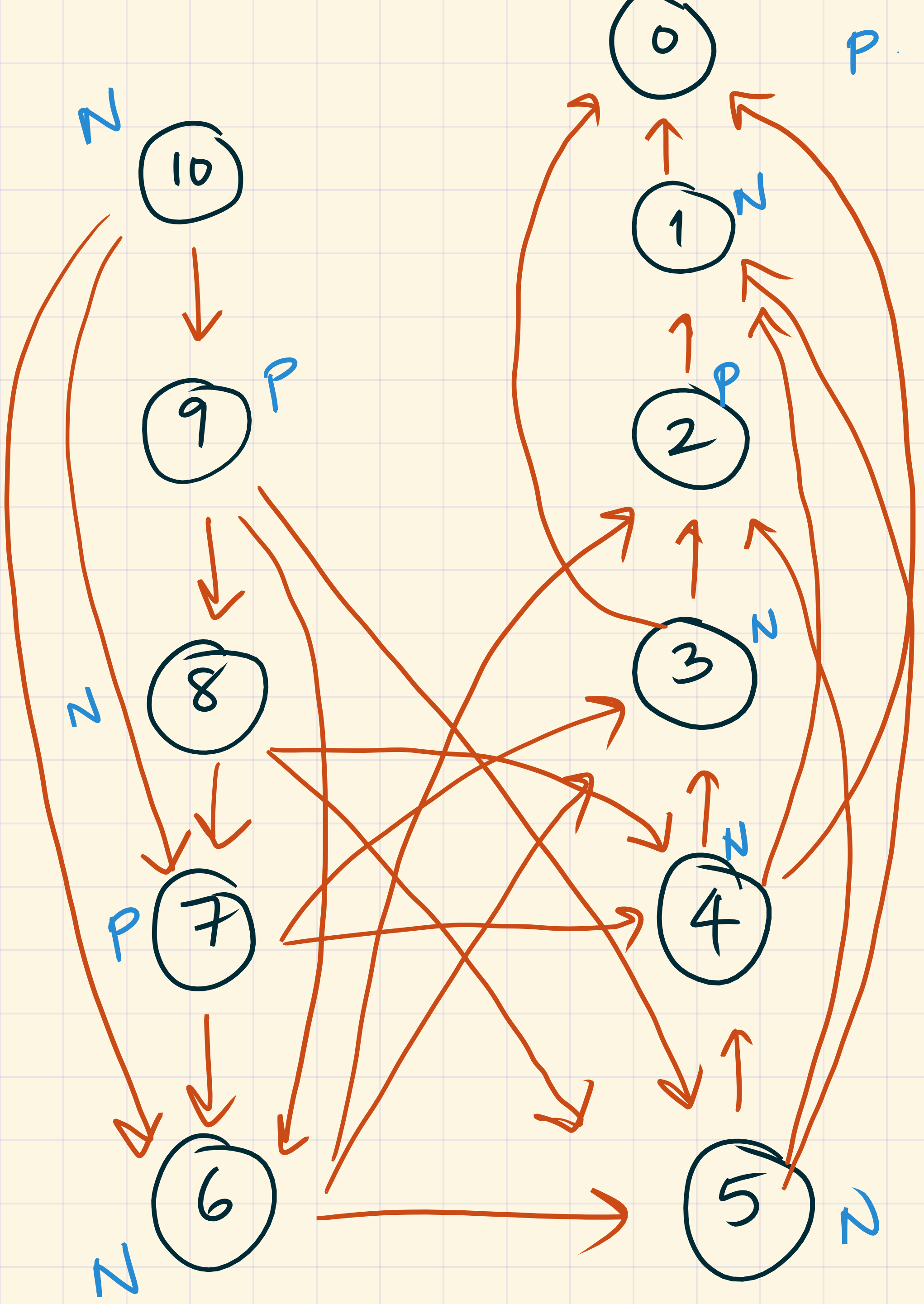
$$10 \to 6 \to 2 \to 1 \to 0$$

$$10 \to 7 \to 6 \to 2 \to 1 \to 0$$

** Use game graph to analyse such games

We say that a game position is an "N" position if it is a winning position for the next player. (That is, the next player has a winning strategy that forces the other player to lose.)

A game state is a "P" position if the previous player has a winning strategy.



Game graph as shown on the left:

We have an edge $a \to b$ if this is a valid move.

1) Any state w/o outgoing arrows is a P position.
2) Any state that points to a P position is an N position.
3) Any state that only points to N positions is a P position.

** <u>Nim</u>

Game state: an ordered finite list of non-negative integers

(or a number of bowls of berries)

A move consists of choosing a bowl and eating some number ≥ 1 of berries from that bowl.

Start : $(4, 5, 2)$

$(2, 5, 2)$

$(1, 5, 2)$

$(1, 0, 2)$

$(1, 0, 1)$ ← P

$(0, 0, 1)$

$(0, 0, 0)$

** Wythoff's game

Game start : (a, b) with $a, b \geq 0$.

Move : Either choose a bowl & eat some (≥ 1) berries from that bowl, or eat the same number (≥ 1) of berries from both bowls