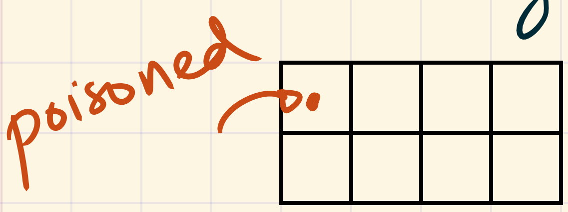
MATH 2301

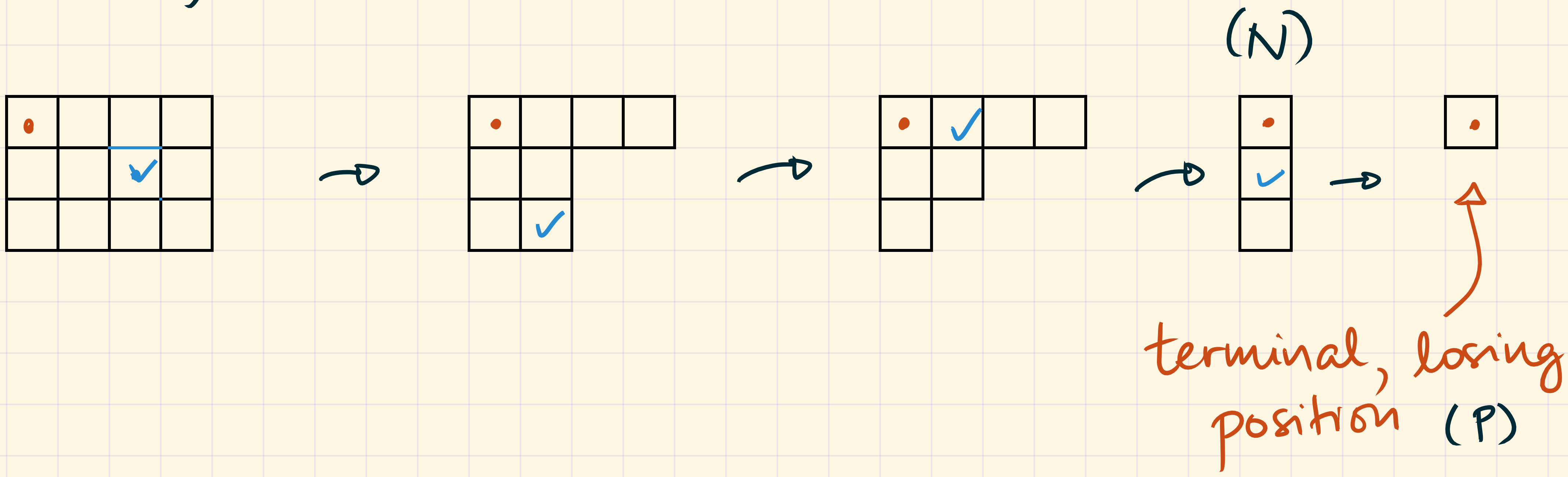
* Further examples of games (impartial, combinatorial)

** Chomp

Starting state: an $m \times n$ bar of chocolate



A move consists of picking a square and eating all the chocolate below & to the right. The person who cannot eat any non-poisoned squares loses. (So 1×1 is a losing position)

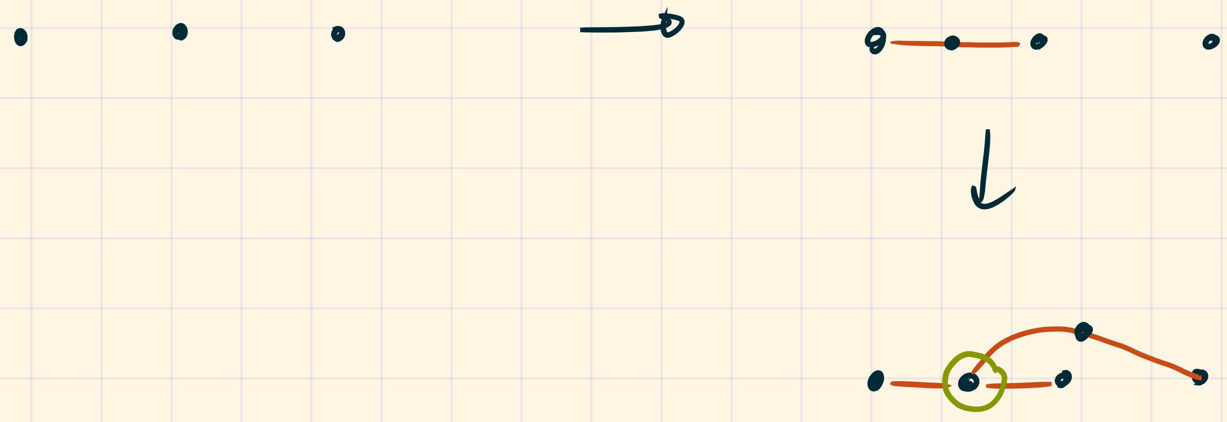


** Sprouts

Starting state = a number of dots

E.g. • • •

Moves: Connect two existing dots by a (curved) line, not passing through any existing dots, and draw a third dot on your segment, making sure that no dot has > 3 incident segments.



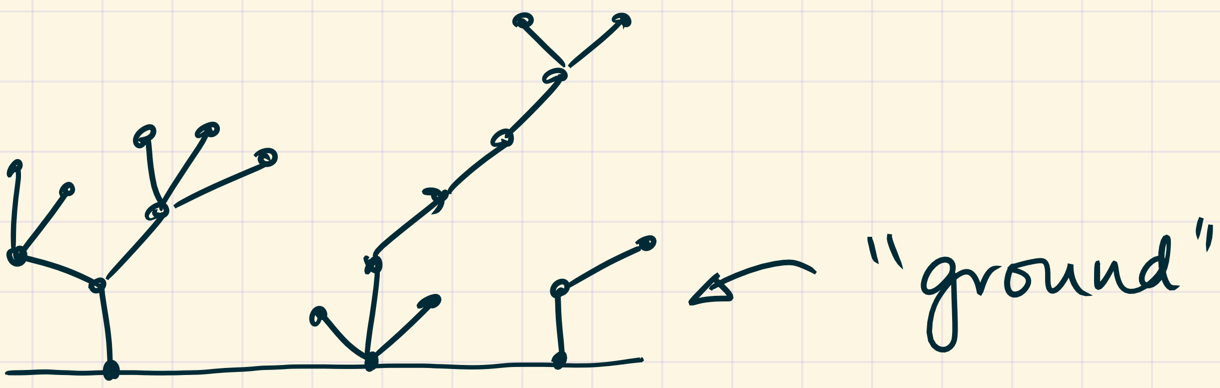
this spot can no longer be connected to anything.

[Look up on wikipedia!]

** Kayles

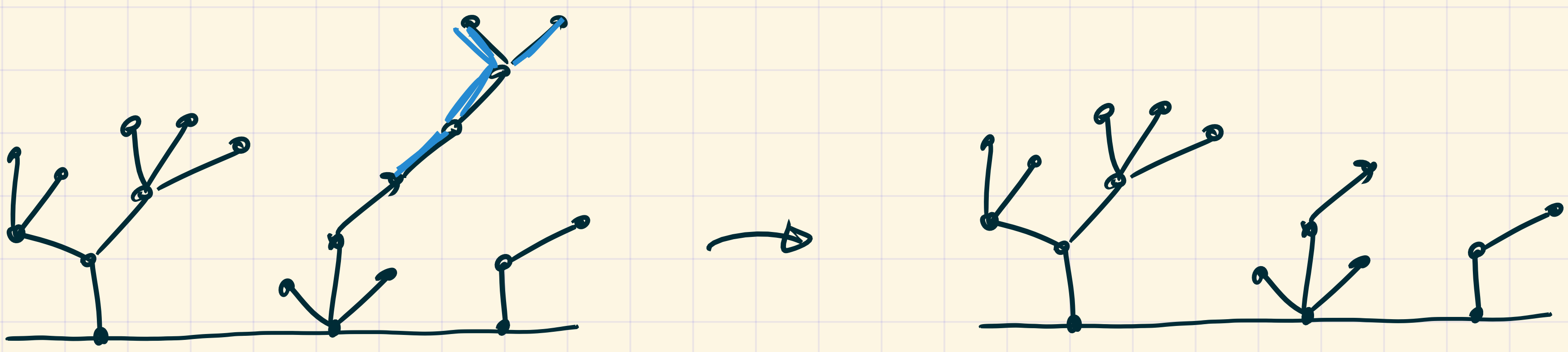
[Look up on wikipedia]

** Hackenbush



State: A number of trees connected to the ground.

Move: Snip a single edge. Anything no longer connected to the ground disappears



** All games above have lots of variants!

** Recap : In principle, we can analyse any of these games!

1) Draw the game graph from the starting position
2) Label the terminal positions as "P"
3) Work backwards, labelling positions as "N" or "P".

But this gets complicated quickly!

** Back to nim

Some special cases

1) A single pile of k_1 berries $(k>0)$ is trivially "N".

2) Two equal piles : (k,k) with $k>0$.
This is a "P" position.
The previous player uses a "mirroring" strategy:
If the next move is $(k-m, k)$,
the move after that should be $(k-m, k-m)$.

Eg. $(4,4) \rightsquigarrow (4,2) \rightsquigarrow (2,2)$

a (k,k) type position again

** Key : (k,k) cannot lead to $(0,0)$

3) Two unequal piles (m, n) with $m \neq n$.

is an "N" position.

Winning strategy : Equalise the piles.

Eg. $(4,7) \rightsquigarrow (4,4)$

** General case (3 or more piles).

E.g. $(2, 1, 3)$ $\leftsquigarrow$ "P"

What are some bad moves for the next player?

- Eating a whole pile (any of the three).
- $(2,1,2)$ is bad.
- $(2, 1, 1)$, $(1,1,3)$ are bad.

$\Rightarrow$ no good moves...

Eg: $(4, 5, 17)$??

Next goal : Find a "value" for each game state, from which the losing positions are clear.