MATH 2301 (S2, 2022)

Course admin

* Check Wattle!
* Workshops in Week 2 +
* Discussion forum (Zulip)
* Office hour (TBA)
* Notes on https://asilata.github.io/ggm/2022

Assessment

* 1 Midterm & 1 final (details forthcoming)
* Assignments (weekly) → sign up to Gradescope
* Reflective check-ins

Outline

* Informal intro to set theory (syntax/language for the course)
* Graph theory
* Posets (partially ordered sets)
* Machines (finite state machines) ↔ regular expressions
* Games (combinatorial games)

Sets

Informally: A set is an unordered collection of distinct objects

$\{p, q, r\}$ or $\{1, 3, 5, 7\}$ but not $\{1, 3, 3, 5, 7\}$

element of this set → $\{q, p, r\}$ = $\{r, q, p\}$ $\{3, 7, 5, 1\}$

Set builder notation: $\{x \in \mathbb{N} \mid x \text{ is even}\} = \{0, 2, 4, 6, 8, \ldots\}$

("such that"; variable; "$\in$" = "in"; $\mathbb{N}$ = natural numbers)

Two sets are equal if and only if they have the same elements.

Rmk

Elements of sets can themselves be sets, e.g. $\{\{1\}, \{2\}\} \neq \{1, 2\}$

Properties

* The empty set = $\{\} = \emptyset$
* Subset: $A \subseteq B$ ($A \subset B$) if every element of A is an element of B. ($\subseteq$ = "is a subset of")

  $A \subsetneq B$ means that B has strictly more elements
* Superset: $B \supseteq A$ if $A \subseteq B$.

Note: $\emptyset \subseteq A$ for every set A.

$\emptyset \subseteq \emptyset$ is also true.

* Union: $A \cup B$ = the set whose elements are the elements of A together with the elements of B (removing repeats)

  E.g. $\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$
* Intersection: $A \cap B$ = set whose elements are those that both in A and in B

  $\{1, 2\} \cap \{2, 3\} = \{2\}$.

  $\{1, 2\} \cap \{3, 4\} = \emptyset$.
* Power set of a set A ($\mathcal{P}(A)$) is the set whose elements are all the subsets of A.

  E.g. $A = \{1, 2\}$

  $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

  $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$
* Cartesian product of two sets A & B ($A \times B$):

  $= \{(a, b) \mid a \in A, b \in B\}$ (ordered pair)

$\{1, 2\} \times \{2, 3\} = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$

$\{1, 2\} \times \emptyset = \emptyset$

$\emptyset \times \{2, 3\} = \emptyset$

Relation (binary relation)

Def: A relation R on sets A and B (in that order) is simply a subset $R \subseteq A \times B$