

MATH 2301 (S2, 2022)

Course admin

- * Check Wattle!
- * Workshops in Week 2 +
- * Discussion forum (Zulip)
- * Office hour (TBA)
- * Notes on <https://asilata.github.io/ggm/2022>

Assessment

- * 1 Midterm & 1 final (details forthcoming)
- * Assignments (weekly) → sign up to Gradescope
- * Reflective check-ins

Outline

- * Informal intro to set theory (syntax/language for the course)
- * Graph theory
- * Posets (partially ordered sets)
- * Machines (finite state machines) ↔ regular expressions
- * Games (combinatorial games)

Sets

Informally: A set is an unordered collection of distinct objects

$\{p, q, r\}$ or $\{1, 3, 5, 7\}$ but not $\{1, 3, 3, 5, 7\}$

element
of this
set

$\{q, p, r\}$

$\{3, 7, 5, 1\}$

"such that"

Set builder notation: $\{x \in \mathbb{N} \mid x \text{ is even}\} = \{0, 2, 4, 6, 8, \dots\}$

variable

"is in"

natural
numbers

Two sets are equal if and only if they have the same elements.

Rmk

Elements of sets can themselves be sets, e.g. $\{\{1\}, \{2\}\} \neq \{1, 2\}$

Properties

* The empty set = $\{\} = \emptyset$

$$A \subset B$$

* Subset: $A \subseteq B$ if every element of A is an element of B.
↑ "is a subset of"

$A \subsetneq B$ means that B has strictly more elements

* Superset: $B \supseteq A$ if $A \subseteq B$.

Note: $\emptyset \subseteq A$ for every set A

$\emptyset \subseteq \emptyset$ is also true.

* Union: $A \cup B$ = the set whose elements are the elements of A together with the elements of B (removing repeats)

E.g. $\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$

* Intersection: $A \cap B$ = set whose elements are those that both in A and in B

$$\{1, 2\} \cap \{2, 3\} = \{2\}$$

$$\{1, 2\} \cap \{3, 4\} = \emptyset$$

* Power set of a set A $\mathcal{P}(A)$ is the set whose elements are all the subsets of A.

E.g. $A = \{1, 2\}$

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

$$\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$$

* Cartesian product of two sets A & B ($A \times B$):

$$= \{ \underbrace{(a, b)}_{\substack{\uparrow \\ \text{ordered} \\ \text{pair}}} \mid a \in A, b \in B \}$$

$$\{1, 2\} \times \{2, 3\} = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$$

$$\{1, 2\} \times \emptyset = \emptyset$$

$$\emptyset \times \{2, 3\} = \emptyset$$

Relation (binary relation)

Def: A relation R on sets A and B (in that order) is simply a subset $R \subseteq A \times B$