MATH 2301 27/07/2022

* Set theory

* Relations

Def: A relation R on sets S & T is a subset

$$R \subseteq S \times T$$

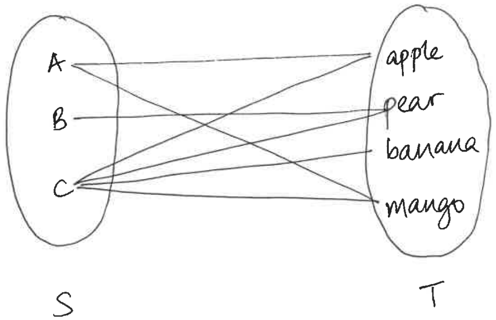
Rmk: This is a binary relation (two sets)
In general, you could have multiple sets $S_1, S_2, \dots, S_n$
& a relation would be a subset $R \subseteq S_1 \times S_2 \times S_3 \times \dots \times S_n$.

Examples

1) ANU MyTimetable enrolment system has lots of relations in the background
such as: $\{$(student, coursecode) | student is enrolled in the course$\}$

2) $S = \{A, B, C\}$ (names of people)

$T = \{$apple, pear, banana, mango$\}$



Corresponding relation R is:
$\{$(A, apple), (A, mango), (B, pear), (C, apple), $\dots\}$

① 2)

Remarks

~~1) Usually, we~~

1) Often in applications, S and T are the same set. In this case, we just say

R is a relation on S.

2) The empty subset $\emptyset \subseteq S \times T$ is a valid relation

3) The full subset $S \times T \subseteq S \times T$ is a valid relation.

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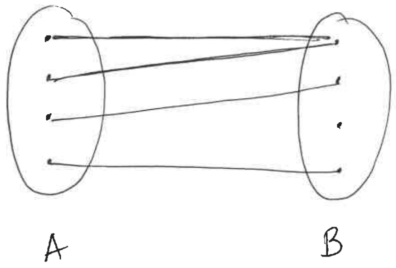
* Functions



A function F is a relation on two sets A & B such that for every element $a \in A$, there is a unique element $b \in B$ such that $(a, b) \in F$.

We often write this as

$F : A \to B$, and say $F(a) = b$ if b is the unique match for $a \in A$.

Other common properties of relations on a single set S

~~1) Reflexivity~~

1) Reflexivity : A relation $R \subseteq S \times S$ is reflexive if $(s,s) \in R$ for every $s \in S$.

2) Symmetry : A relation $R \subseteq S \times S$ is symmetric if whenever $(s,t) \in R$, we also have $(t,s) \in R$.

3) Anti-symmetry : A relation $R \subseteq S \times S$ is anti-symmetric if whenever $(s,t) \in R$ and $s \neq t$, the pair (t,s) is not in R.

~~[Rmk: this means that pairs such as (s,s) are never in an anti-symmetric relation.]~~ See tomorrow's notes for correction.

4) Transitivety : A relation $R \subseteq S \times S$ is transitive if whenever $(s,t) \in R$ and $(t,u) \in R$, then the pair $(s,u) \in R$.

Examples / exercises

$S = \{a, b, c\}$

① $R = \{(a,a), (b,b), (c,c)\}$

- reflexive ✓
- symmetry ✓
- anti-symmetry ~~✗~~ ✓
- transitivity ✓ (vacuously)

② $R = \emptyset$

- reflexivity ✗
- symmetry ✓
- anti-symmetry ✓
- transitivity ✓

$\}$ (vacuously)

③ $R = S \times S$

- reflexivity ✓
- symmetry ✓
- anti-symmetry ✗
- transitivity ✓

Rmk : We say that an "if x then y" statement is vacuously satisfied if the x part is never satisfied.

Exercise : $S = \mathbb{N} = \{0, 1, 2, 3, \ldots\}$

$R = \{(a,b) \in \mathbb{N} \times \mathbb{N} \mid a \neq 0 \text{ and } b \text{ is divisible by } a\}$

E.g. $(2,16) \in R$, but $(3,19) \notin R$.

Which of the four properties does R have?