MATH 2301 — 28/07/2022

From last time (correction)

* Anti-symmetry: A relation $R \subseteq S \times S$ is anti-symmetric if whenever $(s,t) \in R$ and $s \neq t$, we always have $(t,s) \notin R$.

Rmk: This means that pairs of the form (s,s) can be in R.

* Example from last time:

R on $\mathbb{N}$, defined as follows:

$\{(a,b) \mid a \neq 0$ and b is a multiple of $a\}$.

Reflexivity: No, because $(0,0) \notin R$

Symmetry: No, e.g. $(1,12) \in R$ but $(12,1) \notin R$

Anti-symmetry: Yes, because if $(a,b) \in R$ with $a \neq b$ then either $b=0$, or $b>a$. In either case, $(b,a) \notin R$.

Transitivity:

If $(a,b) \in R$ and $(b,c) \in R$: we see that b is a multiple of a, c is a multiple of b, and $a,b \neq 0$; so c is a multiple of $a \Rightarrow$ (implies) $(a,c) \in R$

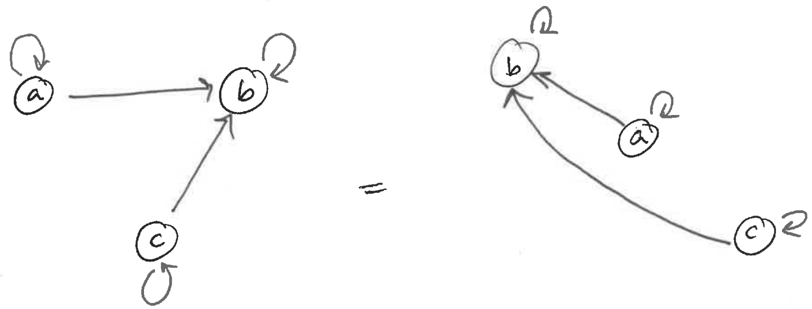
Graphs (Directed graphs)

One way to think about directed graphs is as a tool to visualise relations on a single set.

E.g. $S = \{a,b,c\}$

$R = \{(a,a),(b,b),(c,c),(a,b),(c,b)\}$

Equivalent description: (drawing)

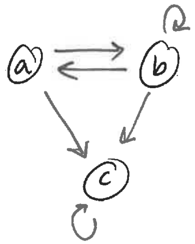


Def: A directed graph consists of a set V (of "vertices"), together with a relation $E \subseteq V \times V$ (E = "edges").

Rmk: ① We don't allow multiple copies of the same edge. (for now).

② Two drawings may correspond to the same graph → this happens if the labels and arrows all match the same relation.

Adjacency matrix of a (directed) graph



First, choose an ordering on the set of vertices. Say a, b, c in that order.

The adjacency matrix (with respect to this ordering) is

$$\begin{array}{c} \\ (a) \\ (b) \\ (c) \end{array} \begin{array}{c} \begin{array}{ccc} (a) & (b) & (c) \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Put 1 in the $(i,j)^{th}$ spot if $(i,j) \in E$. Otherwise, put 0.

What if we had chosen the order (b, a, c)?

$$\begin{array}{c} \\ (b) \\ (a) \\ (c) \end{array} \begin{array}{c} \begin{array}{ccc} (b) & (a) & (c) \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Def: Let $G = (V, E)$ be a directed graph such that we have chosen an ordering on V: $(v_1, v_2, v_3, \ldots)$. The adjacency matrix is a square matrix with rows & columns indexed by the elements of V. The $(v_i, v_j)^{th}$ entry is 1 if $(v_i, v_j) \in E$, and 0 otherwise.

Q: Given an adjacency matrix, can you deduce (just from the properties of the matrix) if the edge relation is reflexive/symmetric/anti-symmetric/transitive?