MATH2301

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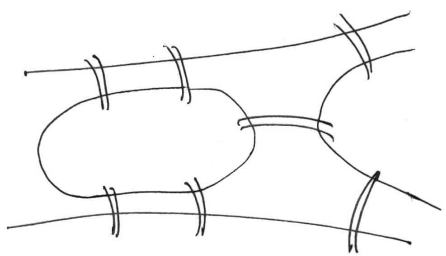
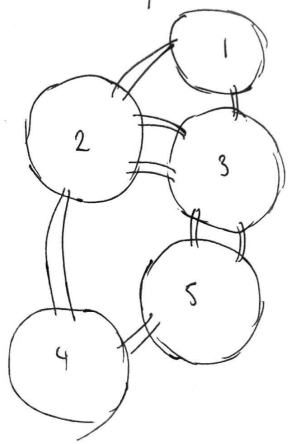
The Bridges of Königsberg



Can you walk around Königsberg crossing each bridge exactly one?

The Roads of Canberraberg

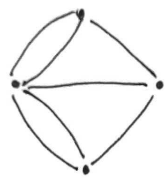


Can you drive around Canberra, going along each road exactly once?

For this problem, we only care about the locations in terms of the connections between them.

Simplified pictures

Königsberg



Canberraberg

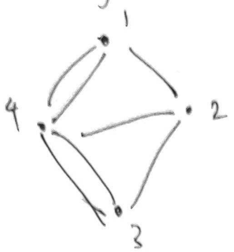
These are examples of graphs.

Dfn: A *graph* is a pair (V, E) consisting of

- V a set of vertices
- E is a set of edges connecting the vertices

These are expressed as pairs of vertices.

Ex: Königberg

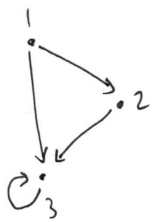


$V = \{1, 2, 3, 4\}$

$E = \{(1,2), (2,1), (2,4), (4,2), (1,4)_1, (1,4)_2, \dots\}$

Dfn: A <u>directed graph</u> is a graph where each edge has a direction.

Ex:



$V = \{1, 2, 3\}$

$E = \{(1,2), (2,3), (1,3), (3,3)\}$

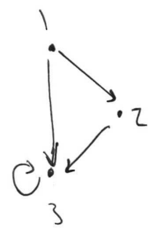
Some examples of graph-theoretic questions.

1. Given two vertices, is there a path between them?
2. If so, how many paths are there?
3. Are there any closed loops?

4. What is the shortest path between vertices?
5. Is it possible to find a path that crosses every edge exactly once?
6. Does there exist a path that visits every vertex exactly once?

We can summarize a graph in a table telling us which vertices have an edge between them.

Ex:



		to		
		1	2	3
from	1	0	1	1
	2	0	0	1
	3	0	0	1

This is a <u>matrix</u>

The ~~graph~~ matrix made this way from the edges of a graph is called the <u>adjacency matrix</u>.

Recall: A $m \times n$ matrix is a table of numbers with m rows and n columns.

$$m\left\{\underbrace{\begin{pmatrix} \vdots & \cdots & \cdots \\ \vdots & \cdots & \\ \vdots & & \end{pmatrix}}_{n}\right.$$

Two matrices that are both $m \times n$ can be added to get another $m \times n$ matrix

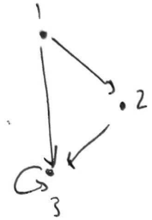
eg. $$\begin{pmatrix} \boxed{1} & 2 \\ 4 & 0 \\ -3 & 1 \end{pmatrix} + \begin{pmatrix} \boxed{0} & 1 \\ 1 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \boxed{1} & 3 \\ 5 & 1 \\ -4 & 1 \end{pmatrix}$$

Two matrices that are $m \times n$ and $n \times k$ can be multiplied to get an $m \times k$ matrix.

eg. $$\underbrace{\begin{pmatrix} 1 & 2 \\ 4 & 0 \\ -3 & 1 \end{pmatrix}}_{3 \times 2} \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}}_{2 \times 2} = \begin{pmatrix} \boxed{2} & 3 \\ 0 & 4 \\ 1 & -2 \end{pmatrix}$$

For any graph, it's adjacency matrix is always a square matrix (ie. $n \times n$). Hence you can multiply it by itself.

Ex:



$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Q: What information about the graph does A^2 contain?