MATH 2301 — 17 Aug 2022

* Midterm exam → end of week 6
  (Look on wattle → online exam)
  (Syllabus → up until the end of week 5)

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* Error from last time:

Transitive closure from Boolean powers

Prop If G is a graph w/ n vertices, and A is an adjacency matrix of G, the adjacency matrix of the transitive closure is:

$$\underbrace{A \vee A^{*2} \vee A^{*3} \vee \dots \vee A^{*n}}_{\text{(entry-wise "or")}}$$

$$A \vee B = \begin{pmatrix} \square & & \\ & & \\ & & \end{pmatrix} \vee \begin{pmatrix} \square & & \\ & & \\ & & \end{pmatrix} = \begin{pmatrix} a_{11} \vee b_{11} & a_{12} \vee b_{12} & \cdots \\ a_{21} \vee b_{21} & & \cdots \\ \vdots & & \cdots \end{pmatrix}$$

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* Rmk: "Path" vs "walk"

Sometimes, a "path" is a ~~walk~~ path through the graph that doesn't repeat vertices, while a walk can repeat vertices

BUT we will not put this restriction.

For me, both paths = walks can repeat vertices

* Partially ordered sets (posets)

Def : A ~~set~~ relation R on a set S is called a partial order if it is:

1. reflexive,
2. anti-symmetric, and
3. transitive

** Notation : Let R be a partial order on S

1. We'll say that S is a poset w.r.t R
2. If $(a,b) \in R$, we'll say $a \preceq_R b$ or just $a \preceq b$

Examples

1. $S = \mathbb{N}$

   $R = \{(x,y) \in \mathbb{N} \times \mathbb{N} \mid x \leq y\}$

   Note : $x \preceq_R y$ if and only if $x \leq y$

2. $S = \mathbb{N}$

   $R = \{(x,y) \in \mathbb{N} \times \mathbb{N} \mid x \geq y\}$

   Note : $x \preceq_R y$ if and only if $x \geq y$

check poset conditions

[In fact, these are total orders, meaning that given $a, b \in S$, either $a \preceq_R b$ or $b \preceq_R a$, or both]

3 Let A be a fixed set

E.g. $A = \{a, b, c\}$

$S = \mathcal{P}(A)$ = set of all subsets

E.g. $\mathcal{P}(A) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \dots \}$

R is the "subset" relation
$\parallel$
$\{ (X, Y) \in \mathcal{P}(A) \times \mathcal{P}(A) \mid X \subseteq Y \}$.

Check that this is a partial order

E.g. $X = \{a, b\}$, $Y = \{a, c\}$

These are "not comparable", meaning that neither (X, Y) nor (Y, X) ~~are~~ is in R.

E.g. $\{a\} \subseteq \{a, b\}$, $\{a\} \subseteq \{a, c\}$

$\{a\} \preceq_R \{a, b\}$, $\{a\} \preceq_R \{a, c\}$.

4 $S = \{1, 2, 3, 4, 5, 6\}$

$R = \{ (a, b) \cancel{\}} \in S \times S \mid$ a is a factor of b $\}$

i.e. $a \mid b$

reads "a divides b"

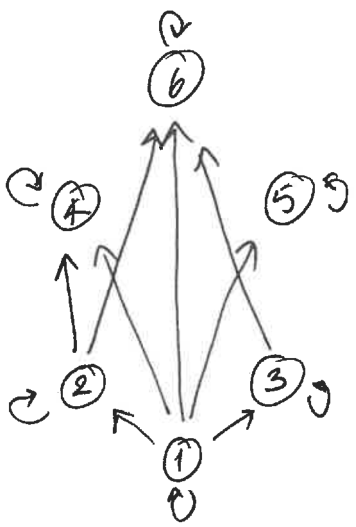
i.e. a is a factor of b

* Hasse diagram of a poset

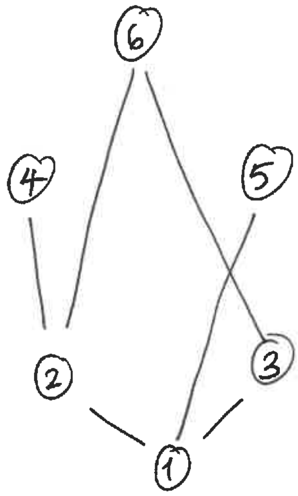
(like the graph of the poset, but with redundant info removed)

E.g. $S = \{1, 2, 3, 4, 5, 6\}$

$R = \{(a,b) \in S \times S \mid a \mid b\}$



Graph, where each non-loop edge "goes up"



Hasse diagram

Graph → Hasse diagram

1. Draw it so that all non-loop edges go upwards
2. Delete all self-loops
3. Delete all edges implied by transitivity.
4. Remove all arrowheads, with the understanding that the lines are oriented upwards

[Reverse these steps to go from a Hasse diagram to the graph drawing]

E.g.

1. $S = \{1, 2, 3, 4\}$

   $\leq$ relation

(The Hasse diagram of any total order is a straight line.)

2. $A = \{a, b, c\}$

   $S = P(A)$, with the subset relation