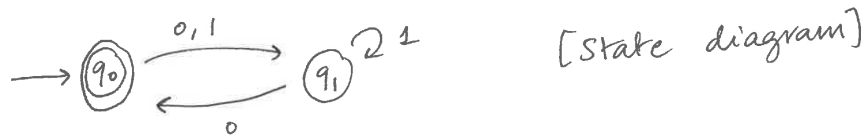


* Deterministic finite automata (DFAs)



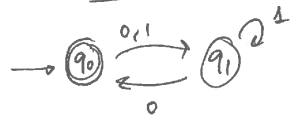
** Def: A DFA consists of the following:

- (1) An alphabet Σ
- (2) A finite set Q of "states"
- (3) A "starting state" $q_0 \in Q$ [denoted $\rightarrow q_0$ by hanging incoming arrow]
- (4) A set $A \subseteq Q$ of "accepting states" [denoted by a double circle]
- (5) A transition function

$$\delta: Q \times \Sigma \rightarrow Q$$

↑ state you're at
↑ letter you're reading
↑ the state you end up in

** Example:



$\Sigma = \{0, 1\}$
 $Q = \{q_0, q_1\}$
 start state = q_0
 $A = \{q_0\}$

Q	Σ	output
q_0	0	q_1
q_0	1	q_1
q_1	0	q_0
q_1	1	q_1

** Reading strings (example from earlier)

$w = 1101$

Read w letter-by-letter (left to right), and keep track of your state by following δ , or the transition arrows.

At the end, we say that the machine accepts w if we end at an accepting state, or rejects w otherwise.

Progression:

- ① start at q_0 , read 1 $\rightarrow q_1$
- ② from q_1 , read 1 $\rightarrow q_1$
- ③ from q_1 , read 0 $\rightarrow q_0$
- ④ from q_0 , read 1 $\rightarrow q_1$
- ⑤ string ended, $q_1 \notin A \rightarrow$ REJECT.

(Some) Accepted strings: 1000, 00, ϵ , 110, ...

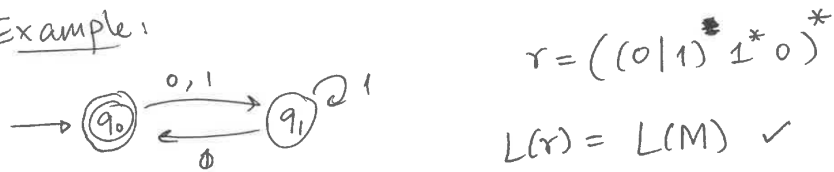
(some) rejected strings: 1, 11, 100, ...

** Language of a DFA

Let M be a DFA. The language of M , denoted $L(M)$, is precisely the set of strings in Σ^* that are accepted by M .

* Question: Given a DFA M , is there a regex r such that $L(M) = L(r)$? Or vice-versa?

** Example:



* Let us try to build a machine (DFA) M corresponding to any given regex r , i.e., a machine M with $L(M) = L(r)$

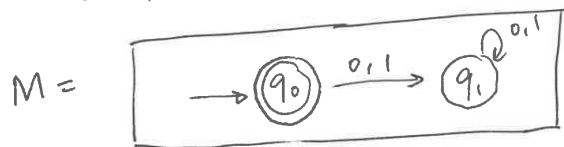
Do this constructor-by-constructor. (Fix $\Sigma = \{0,1\}$)

(1) $r = \emptyset$, $L(r) = \emptyset$



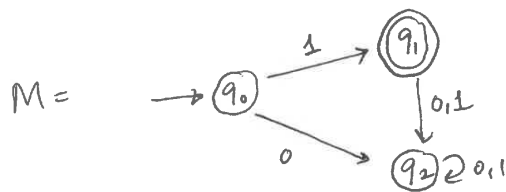
(there are other options: just make every state rejecting!)

(2) $r = \epsilon$, $L(r) = \{\epsilon\}$



(or other options...)

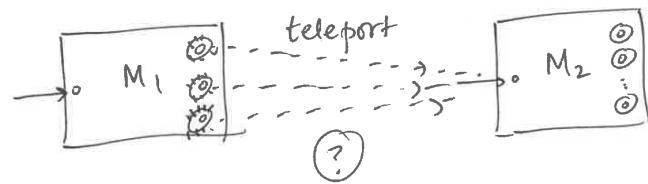
(3) $r = a$ for $a \in \Sigma$, e.g. $r = 1$, $L(r) = \{1\}$



[similar construction for $r = 0$]

(4) $r = r_1 r_2$, $L(r) = L(r_1) \circ L(r_2)$

Suppose M_1 & M_2 are DFAs such that $L(M_1) = L(r_1)$, $L(M_2) = L(r_2)$



Next time ...