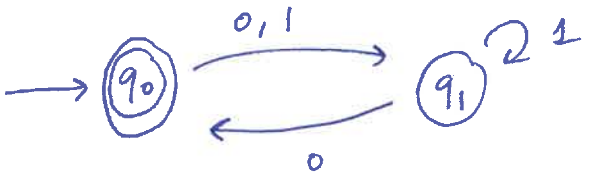
MATH 2301

26 Sept 2022

* Deterministic finite automata (DFAs)



[state diagram]

Def: A DFA consists of the following:

(1) An alphabet Σ

(2) A finite set Q of "states"

(3) A "starting state" $q_0 \in Q$ [denoted $\rightarrow$ q_0 by hanging incoming arrow]

(4) A set $A \subseteq Q$ of "accepting states" [denoted by a double circle]

(5) A transition function

$$\delta : \underset{\substack{\uparrow \\ \text{state} \\ \text{you've at}}}{Q} \times \underset{\substack{\uparrow \\ \text{letter} \\ \text{you're} \\ \text{reading}}}{\Sigma} \longrightarrow Q$$

$\leftarrow$ the state you end up in

Example:

$\Sigma = \{0,1\}$

$Q = \{q_0, q_1\}$

start state $= q_0$

$A = \{q_0\}$

δ:

Q	Σ	output
q_0	0	q_1
q_0	1	q_1
q_1	0	q_0
q_1	1	q_1

** Reading strings (example from earlier)

$w = 1101$

Read w letter-by letter (left to right), and keep track of your state by following δ, or the transition arrows.

At the end, we say that the machine accepts w if we end at an accepting state, or rejects w otherwise.

Progression:

1. start at q_0, read $1 \to q_1$
2. from q_1, read $1 \longrightarrow q_1$
3. from q_1, read $0 \longrightarrow q_0$
4. from q_0, read $1 \longrightarrow q_1$
5. string ended, $q_1 \notin A \to$ REJECT.

(Some) Accepted strings: 1000, 00, ε, 110, ...

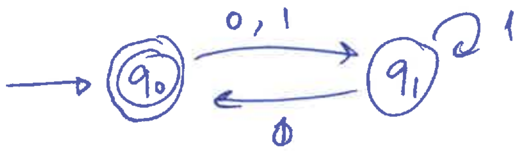
(some) rejected strings: 1, 11, 100, ...

** Language of a DFA

Let M be a DFA. The language of M, denoted $L(M)$, is precisely the set of strings in Σ^* that are accepted by M.

* Question: Given a DFA M, is there a regex r such that $L(M) = L(r)$? Or vice-versa?

** Example:



$r = ((0|1)^* 1^* 0)^*$

$L(r) = L(M)$ ✓

* Let us try to build a machine (DFA) M corresponding to any given regex r, i.e., a machine M with $L(M) = L(r)$

Do this constructor-by-constructor. (Fix $\Sigma = \{0, 1\}$)

(1) $r = \phi$, $L(r) = \phi$

$M =$ 

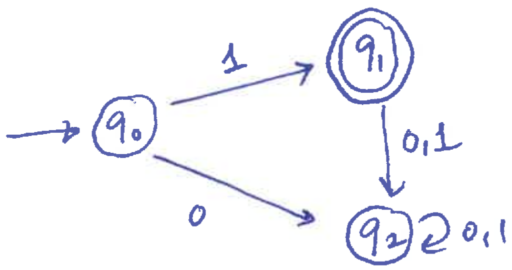
(there are other options: just make every state rejecting!)

(2) $r = \varepsilon$, $L(r) = \{\varepsilon\}$

$M =$ 

(or other options..)

(3) $r = a$ for $a \in \Sigma$, e.g $r = 1$, $L(r) = \{1\}$

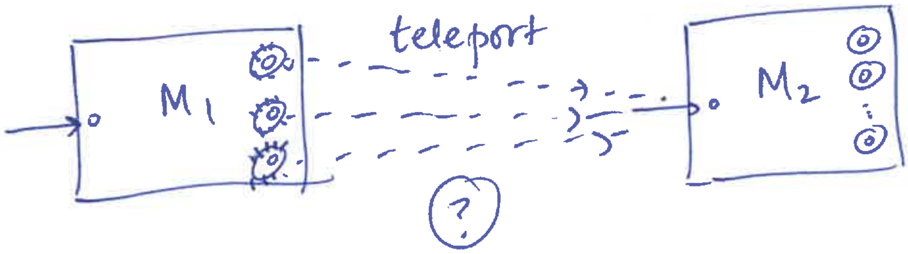
$M =$ 

[similar construction for $r = 0$]

(4) $r = r_1 r_2$, $L(r) = L(r_1) \circ L(r_2)$

Suppose M_1 & M_2 are DFAs such that

$L(M_1) = L(r_1)$, $L(M_2) = L(r_2)$



Next time ...