MATH 2301

28 Sep 2022

* Last time: Constructed DFAs for

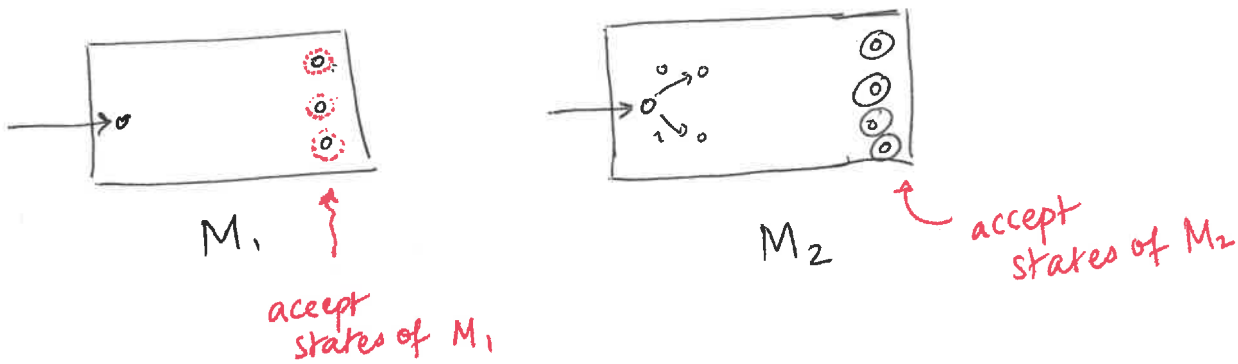
$r = \phi$, $r = \varepsilon$, $r = a$ for $a \in \Sigma$

* Today: Do the rest

(4) $r = r_1 r_2$

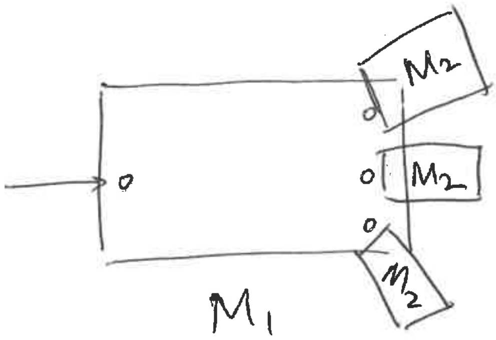
Suppose we have DFAs M_1 & M_2, such that $L(M_1) = L(r_1)$, $L(M_2) = L(r_2)$

We want M such that $L(M) = L(r)$
$= L(r_1) \circ L(r_2)$

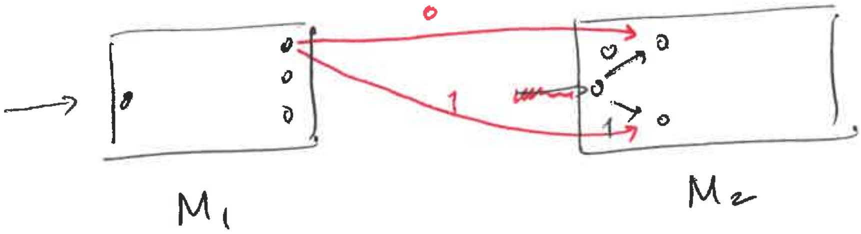


You sort of want to "teleport" from the accept states of M_1 to the start state of M_2 but this is not allowed in a DFA.

Possible solution: attach a copy of M_2 to every accept state of M_1, making that state the start state of M_1



Alternatively, connect each accepts state of M_1 via a letter a to wherever the start state of M_2 would have gone via a.



Note: It's not clear that these are DFAs :(

[We need to fix this ... we'll have to come back to this.]

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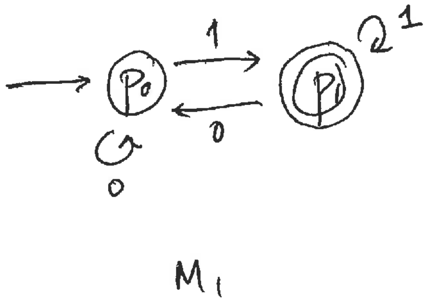
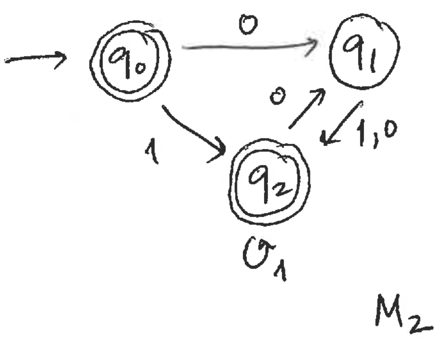
(5) $r = r_1 \mid r_2$, $L(r) = L(r_1) \cup L(r_2)$

Suppose we have machines M_1, M_2 such that $L(r_1) = L(M_1)$, $L(r_2) = L(M_2)$

Example:

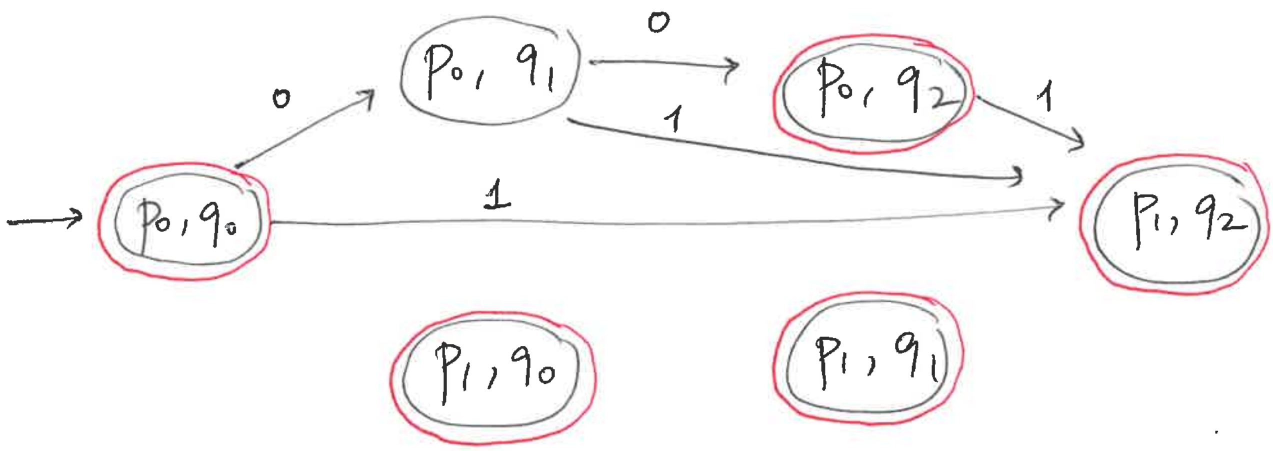




$w = 11011$

Build M with a "product" construction.

States of M = product $P \times Q$



(fill in ...) → exercise

Start state is (p_0, q_0)

Transition function $(P \times Q) \times \Sigma \xrightarrow{\delta} (P \times Q)$

$$\delta((p,q), a) = (\delta_1(p,a), \delta_2(q,a))$$

transition fns of M_1 & M_2

Accept states $= \{(p,q) \in P \times Q \mid$ either p is accepting for M_1, or q is accepting for M_2, or both$\}$

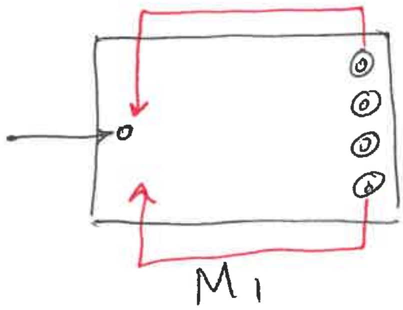
You get a DFA! M

$L(M) = L(M_1) \cup L(M_2)$.

6. $r = (r_1)^*$, $L(r) = L(r_1)^*$

Suppose we have M_1, such that

$L(M_1) = L(r_1)$



want to teleport from the accepting states to the start state.

???

Not clear how to successfully do this !!

[Let's come back to this problem later.]

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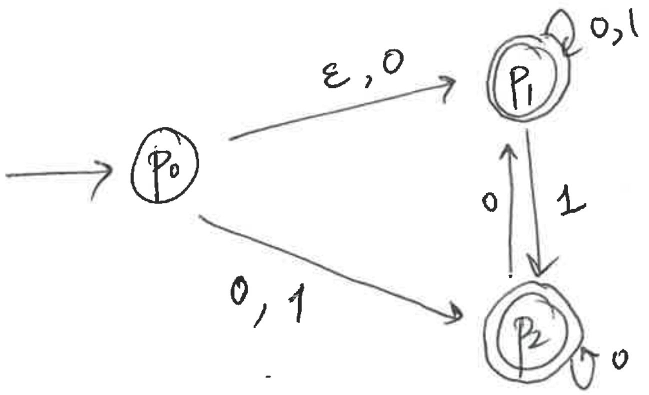
* Non-deterministic finite automata (NFAs)

Think of these as DFAs where you have choices of where to go next.



← NFAs look like this.