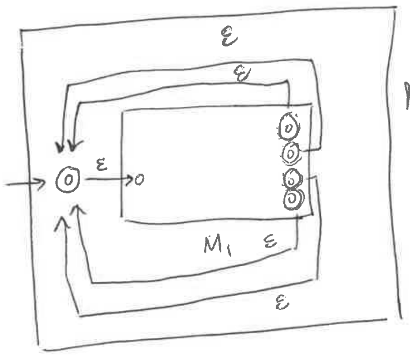
MATH 2301 — 05 Oct 2022

** Regex → NFA

Construction of NFA for $r = (r_1)^*$, given M_1 that recognises r_1

M_1

$L(M_1) = L(r_1)$



M

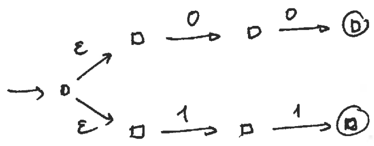
M accepts anything that successfully goes through M_1 0 or more times, ending up at one of the accept states marked.

Example: $(00|11)0 = r$

← NFA recognising $r = 00$

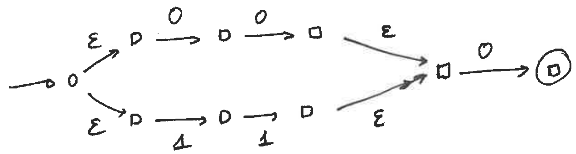
← NFA recognising $r = 11$

← NFA recognising $r = 0$



recognises $(00|11)$

Finally:



NFA recognises exactly $r = (00|11)0$.

Upshot: Given any regex r, there is an NFA M such that $L(r) = L(M)$.

Q: What about DFAs?

Goal: Understand the relationship between what NFAs can do and what DFAs can do.

* Early observation:

Proposition: Let M be an NFA which possibly has ε-arrows. Then there is an equivalent NFA M' which has no ε-labelled arrows, i.e. $L(M') = L(M)$

Proof through an example (proof idea):

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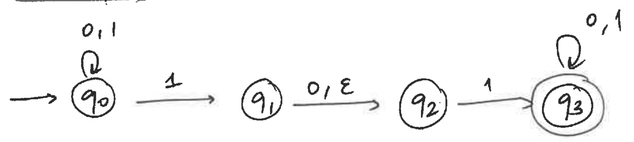
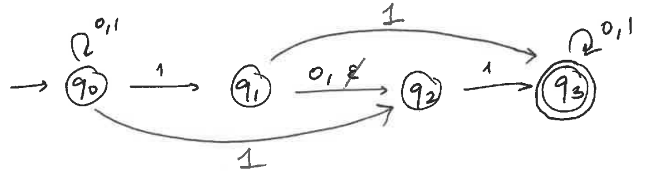
** Example





Idea: Any time there are arrows

$A \xrightarrow{\varepsilon} B \xrightarrow{a} C$ or

$A \xrightarrow{a} B \xrightarrow{\varepsilon} C$, we add in a direct

arrow $A \xrightarrow{a} C$, and continue doing this.

At the end, we'll have an equivalent NFA without ε.

Note: The new machine M' has no ε-labels. The transition function of M' can be thought of as a function

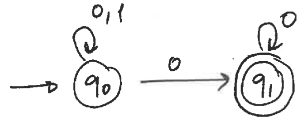
$$\delta : Q \times \Sigma \to \mathcal{P}(Q)$$

Theorem: Given any NFA M, there is an equivalent DFA M', such that $L(M) = L(M')$.

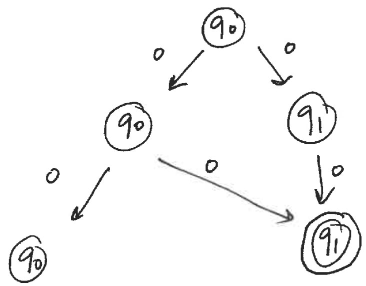
Pf (with an example)

Let M be an NFA. Let us assume further that M does not have any ε-arrows (by previous Prop.)

Eg.



Example calculation tree for $w = 00$

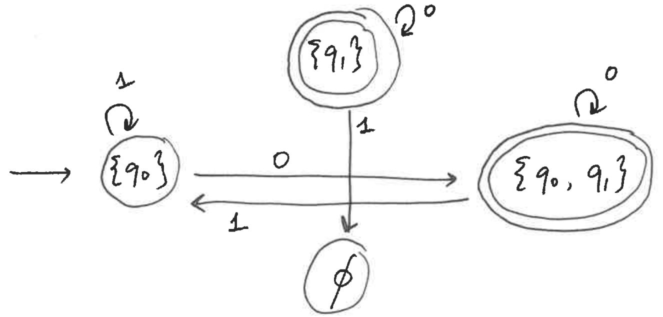


ACCEPT.

Calculation tree moves from a subset of Q to another subset of Q, after reading a single letter.

Let us construct an equivalent DFA. M'

- State set of $M' = \mathcal{P}(Q)$
  $[\{\phi, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}]$
- Start state $= \{q_0\}$
- Accepting states $= \{B \subseteq Q \mid B$ contains an accepting state of $M\}$



Transition function should "follow the calculation tree"

[More tomorrow ...]