MATH 2301

13/10/2022

* Games [Impartial combinatorial games]

An impartial combinatorial game is:

- played between two players;
- turn-based (players take turns).
- There is a game state before every turn, and the possible moves depend only on this state (not on who goes next).
- It is perfect information (no secrets)
- It is deterministic (no dice rolls / luck)
- The game ends if the next player has no move to make.
- There are finitely many possible game states (reachable) after the first state ⇒ each game stops.
- No backtracking, i.e. no directed cycles of moves between game states.

* Example: Subtraction game

Fix some finite set $S = \{1, 3, 4\}$

Start state = some $n \geq 1$

Move = subtract exactly one $k \in S$ from n, to produce $(n-k) \geq 0$

Let's start with $n = 10$, $S = \{1, 3, 4\}$

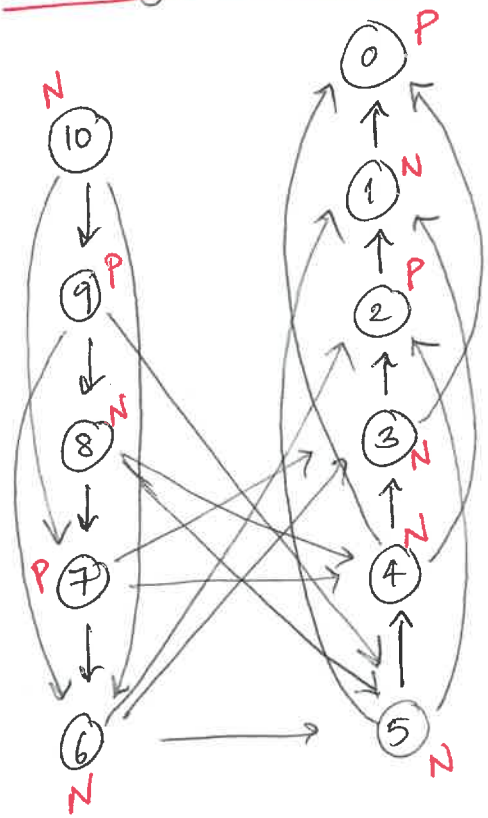
$10 \rightarrow 9 \rightarrow 6 \rightarrow 3 \rightarrow 0$

$25 \rightarrow 21 \rightarrow 18 \rightarrow 17 \rightarrow 16 \rightarrow 13 \rightarrow 9 \rightarrow 8$

$0 \leftarrow 1 \leftarrow 2 \leftarrow 6 \leftarrow 7$

Goal: Who wins?? Which player (1 or 2) has a winning strategy for a given start state?

* Use game graph



Label each position (state) as "N" or "P"

"N": winning for the next player

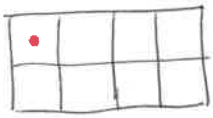
"P": win for previous player

- Terminal positions (those with no outgoing arrows) are P
- Anything with an arrow to a P-position is an N
- If a state only has edges to N-positions, then it is a P-state.

How to win if you are at an N-state?

→ Move to a P-state.

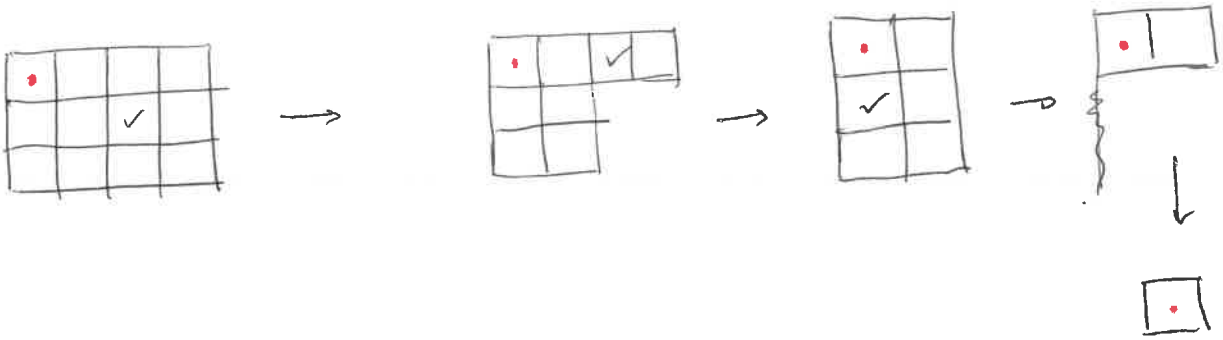
* Chomp



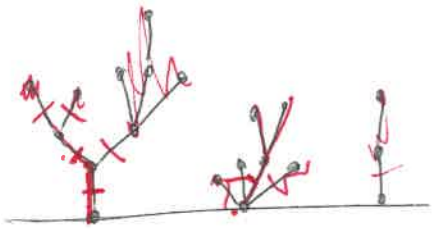
An mxn bar of chocolate
Top left square is poisoned & inedible.

A move : pick a non-poisoned square, and eat everything in the rectangle to the right & below (eat all squares below + right)

The person who cannot make a move loses.



* Hackenbush



← ground

Game state = forest as shown.

Move = chop off a single brach. Anything not connected to the ground falls off.

* Look up : Kayles, Sprouts, Wythoff's game, ...
(on Wikipedia)