

Math 2301 (Games, Graphs, Machines) 24/7/2023

* Admin

- Check Wattle!
- Sign up to Zulip
- Course reps?
- Office hour (TBA)
- Gradescope (?)
- Notes on <https://asilata.github.io/ggm/2023>
- ~~Week~~

My name: Asilata Bapat
 • HN 4.84
<https://asilata.github.io>
 asilata.bapat@anu.edu.au

* Assessment

- Weekly quizzes & final → **** READ CAREFULLY ON WATTLE ****
- Assignments (~ weekly) → Due on Fridays at 11:59pm.
- Reflective check-ins → Due Sunday nights
- Workshops → Start in Week 2.

* Outline

- Informal intro to set theory
- Graphs → Adjacency matrices, several versions
 ↳ Graph colouring
- Posets (partially ordered sets)
- Machines → finite state machines (finite automata)
 ↳ regular expressions
- Games → combinatorial games.

* Sets

Informally, a set is an unordered collection of "elements", without duplicates.

Eg. $\{p, q, r\}$ or $\{1, 3, 5, 7\}$

Set builder notation

$\{x \in \mathbb{N} \mid x \text{ is even}\}$

↑ variable ↓ in ↓ some other set ↘ x satisfies some property.

"such that"

Recall: $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$

* Two sets are considered equal if they have the same elements.

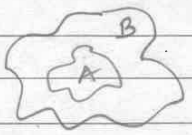
$\{1, 2, 3\} = \{3, 2, 1\}$
a unique

* There is an "empty set", written as \emptyset or $\{\}$

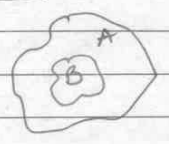
* Sets can contain other sets
 Eg. $\{\{1, 2\}, \{3, 4\}\} \neq \{1, 2, 3, 4\}$
 $\{\emptyset\} \neq \emptyset$.

Properties

* Subset: $A \subset B$ (or $A \subseteq B$) if every element of A is also an element of B .

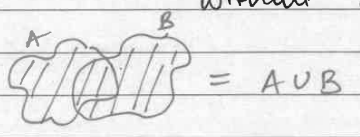


* Superset: $A \supset B$ ($A \supseteq B$) if $B \subseteq A$.

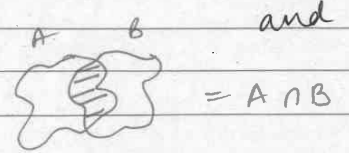


Note: $\emptyset \subseteq A$ for every set A .
(Also, $\emptyset \subseteq \emptyset$)

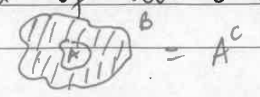
* Union: $A \cup B$ = the set whose elements are those of A together with those of B , without duplicates.



* Intersection: $A \cap B$ = the set whose elements are those that appear are elements of both A and B .



* Complement: If $A \subseteq B$, then A^c (in B) is the set of all elements of B that are not in A .



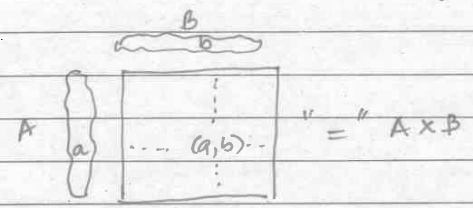
* Product (or Cartesian product)

If A, B are two sets then $A \times B$ (product) is the set

$$\{ (a, b) \mid a \in A, b \in B \}$$

↑
ordered pair

E.g. $\{1, 2\} \times \{1, 5\} = \{(1, 1), (1, 5), (2, 1), (2, 5)\}$



↓
Not the same as $(1, 2)$.

* Power set of a set

If A is a set, the power set of A ($P(A)$) is the set whose elements are all of the subsets of A .

E.g. $A = \{1, 2\}$

$$P(A) = \{ \{1\}, \{2\}, \{1, 2\}, \emptyset \}$$

Note: If A has n elements, the $P(A)$ has 2^n elements.

E.g.

- $\emptyset \times \{a, b\} = \emptyset, \{a, b\}, \{\emptyset\}$

- $P(\emptyset) = \{\emptyset\}$