Math 2301 (Games, Graphs, Machines) 24/7/2023

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* Admin

- Check Wattle!
- Sign up to Zulip
- Course reps?
- Office hour (TBA)
- Gradescope (?)
- Notes on https://asilata.github.io/ggm/2023
- ~~Week~~

* Assessment

- Weekly quizzes & final → ** READ CAREFULLY ON WATTLE
- Assignments (~ weekly) → Due on Fridays at 11:59 pm.
- Reflective check-ins → Due Sunday nights
- Workshops → Start in Week 2

* Outline

- Informal intro to set theory
- Graphs → Adjacency matrices, several versions
  → Graph colouring
- Posets (partially ordered sets)
- Machines → finite state machines (finite automata)
  → regular expressions
- Games → combinatorial games.

* Sets

Informally, a set is an unordered collection of "elements", without duplicates.

E.g.

$\{p, q, r\}$ or $\{1, 3, 5, 7\}$

Set builder notation

$\{x \in \mathbb{N} \mid x \text{ is even}\}$

- x: variable
- $\in$: in
- $\mathbb{N}$: some other set
- $\mid$: "such that"
- x is even: x satisfies some property.

Recall: $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$

* Two sets are considered equal if they have the same elements

$\{1, 2, 3\} = \{3, 2, 1\}$

* There is a unique "empty set", written as $\emptyset$ or $\{\}$

* Sets can contain other sets

E.g. $\{\{1, 2\}, \{3, 4\}\} \neq \{1, 2, 3, 4\}$

$\{\emptyset\} \neq \emptyset$.

Properties

* **Subset**: $A \subset B$ (or $A \subseteq B$) if every element of A is also an element of B.

* **Superset**: $A \supset B$ ($A \supseteq B$) if $B \subseteq A$.

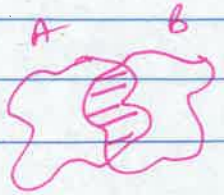
Note: $\phi \subseteq A$ for every set A.

(Also, $\phi \subseteq \phi$)

* **Union**: $A \cup B$ = the set whose elements are those of A together with those of B, without duplicates.

$= A \cup B$

* **Intersection**: $A \cap B$ = the set whose elements are those that ~~appear~~ are elements of both A and B.



$= A \cap B$

* **Complement**: If $A \subseteq B$, then A^c (in B) is the set of all elements of B that are not in A

$= A^c$

* **Product (or Cartesian product)**

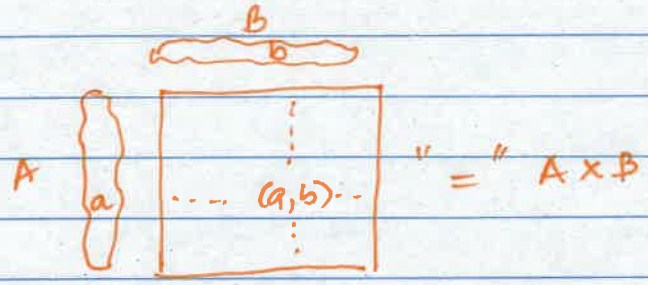
If A, B are two sets then $A \times B$ (product) is the set

$$\{ (a,b) \mid a \in A, b \in B \}$$

(ordered pair)

E.g. $\{1,2\} \times \{1,5\} = \{(1,1), (1,5), (2,1), (2,5)\}$

Not the same as $(1,2)$.



* **Power set of a set**

If A is a set, the power set of A ($\mathcal{P}(A)$) is the set whose elements are all of the subsets of A.

E.g. $A = \{1,2\}$

$$\mathcal{P}(A) = \{ \{1\}, \{2\}, \{1,2\}, \phi \}$$

Note: If A has n elements, the $\mathcal{P}(A)$ has 2^n elements.

E.g.

- $\phi \times \{a,b\} = \phi$, ~~$\{a,b\}$~~, ~~$\{\phi\}$~~
- $\mathcal{P}(\phi) = \{\phi\}$