MATH 2301

26 July 2023

[These are catch-up notes. The AV did not work that day, so I have transcribed the main points of the content here]

* **Last time**: Set theory basics.

  (see https://asilata.github.io/ggm/2023/ggm.pdf)

* **Today**: Relations

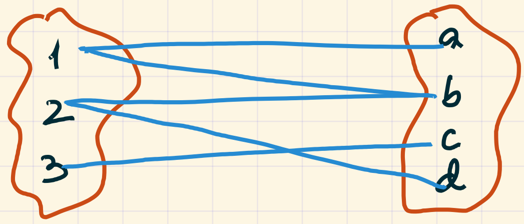
  Def: A (binary) relation on sets S & T is a subset $R \subseteq S \times T$.

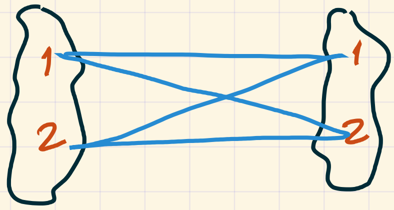
  $\leftarrow$ Product

Note:

- The empty relation $\phi \subseteq S \times T$ is a valid relation
- The full relation $S \times T \subseteq S \times T$ is a valid relation.
- We usually take $S = T$, in which case we say R is a relation on S.
- An n-ary relation on $S_1, S_2, \ldots S_n$ is a subset $R \subseteq S_1 \times S_2 \times \cdots \times S_n$.

* Examples

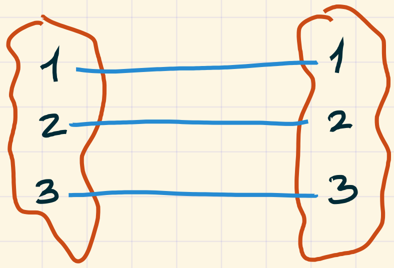


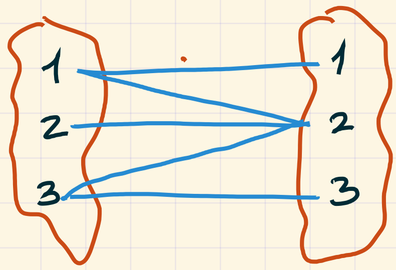


Basic properties of relations

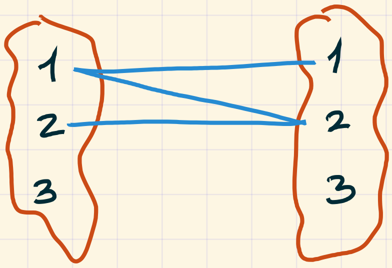
* Reflexivity : R is reflexive if for every $a \in S$ we have $(a,a) \in R$.

E.g.



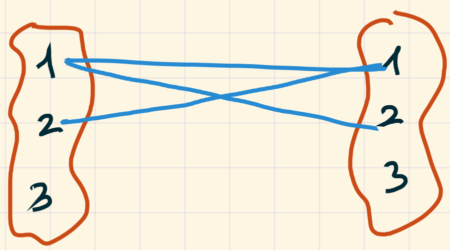


Non-example



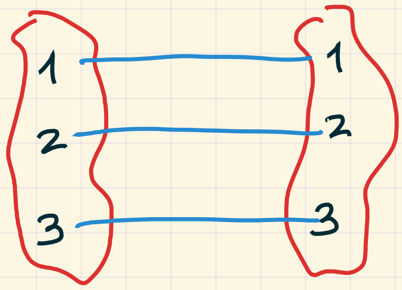
* Symmetry : R is symmetric if whenever $(a,b) \in R$ we have $(b,a) \in R$.

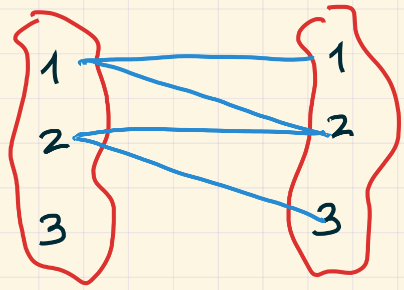
E.g.



* <u>Anti-symmetry</u>: R is anti-symmetric if for every $(a,b) \in R$ with $a \neq b$, the pair (b,a) is not in R.

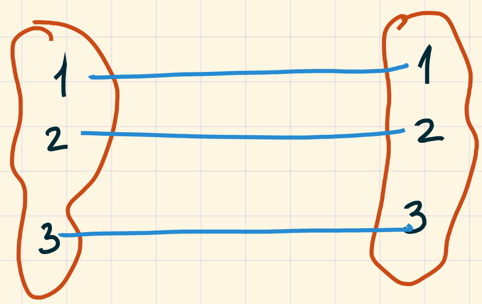
<u>E.g.</u>

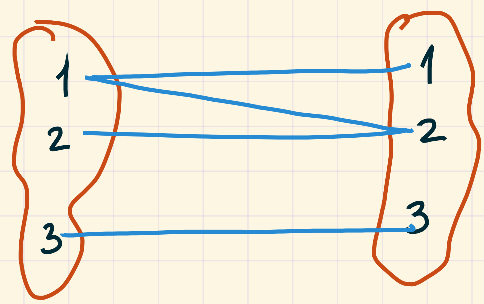


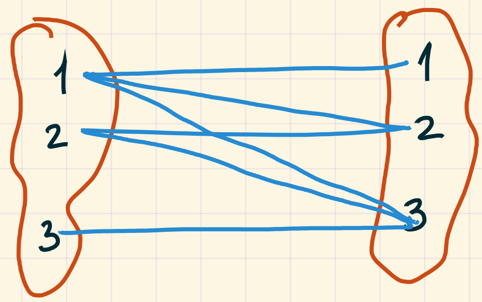


* <u>Transitivity</u>: R is transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ we have $(a,c) \in R$.

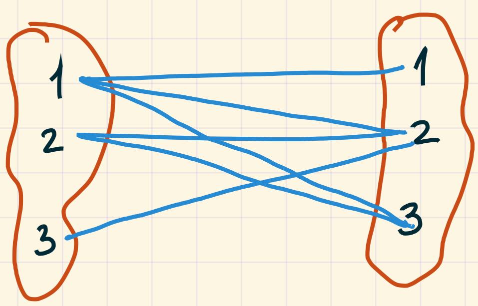
<u>E.g.</u>







<u>Nonexample</u> [why? $\to$ exercise]



* Functions

Def: A relation $R \subseteq S \times T$ is a function if for every $a \in S$ there is a unique $b \in T$ such that $(a, b) \in S$.

E.g.