Math 2301 28/07/2023

* Admin : - Gradescope coming soon!
  - HW 1 will release today on Wattle, due at 11:59 pm on Friday 04 Aug
  - Workshops start next week!
  - Worksheet available this weekend on Wattle. (Look at the problems in advance ~ 1 hour on your own.)
  - Office hrs starting next week Wed & Fridays.

* Last time : - Relations + properties
  - Directed graphs (to visualise)

  (Come back to graphs later in more detail.)

* Today : Equivalence relations

Defn: A relation R on a set S is called an equivalence relation if it satisfies all of the following:

① Reflexivity → every (a,a) is in R

② Symmetry → $(a,b) \in R \Rightarrow (b,a) \in R$

i.e. if $(a,b) \in R$ then $(b,a) \in R$

③ Transitivity. → If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$.

* Examples and non-examples

- $S = \{1,2,3\}$

① $S = \{1,2,3\}$

$R \subseteq S \times S$

$$R = \{(a,b) \in S \times S \mid a = b\} = \{(1,1),\ (2,2),\ (3,3)\}$$

Recall from Wed

→ reflexivity, symmetry, transitivity.

so, it is an equivalence relation.

② $S = \mathbb{Z}$ = set of integers.

$$R = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} \mid a+b \text{ is even.}\}$$

$$= \{(0,0),\ (1,-1),\ (1,5),\ (2,24),\ (-3,15),\ \dots\}$$

✓ reflexivity? yes, $(a,a) \in R$ b/c $a+a = 2a$ (even).

✓ symmetry? yes, if $a+b$ is even then $b+a$ is even.

✓ transitivity? suppose $(a,b) \in R$ and $(b,c) \in R$.

i.e. $a+b$ is even, and $b+c$ is even.

$\Rightarrow (a+b)+(b+c)$ is even.

$\Rightarrow (a+c)+2b$ is even.

$\Rightarrow (a+c)$ is even.

$\Rightarrow (a,c) \in R$.

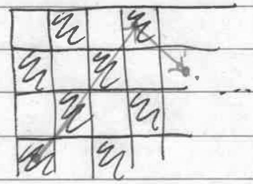
so, it is transitive.

so it is an equiv. relation.

3. $S = \mathbb{Z}$

$R = \{(a,b) \in \mathbb{Z}\times\mathbb{Z} \mid a+b \text{ is odd}\}$.

(Not an equivalence relation.)
→ Not reflexive!

4. 

(8×8) chessboard.

S = set of squares on this chessboard.

$R = \{(a,b) \in S\times S \mid$ you can get to b from a by a sequence of bishop moves $\}$ (possibly (no moves)).

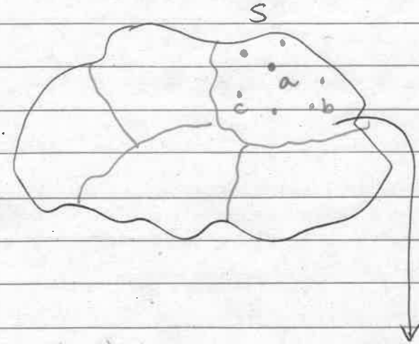
reflexivity? yes

symmetry? yes (move backwards)

transitivity? Let $(a,b)\in R$, $(b,c)\in R$. go from $a \to b$, then $b \to c$, gives you a sequence $a \to c$, so $(a,c)\in R$.

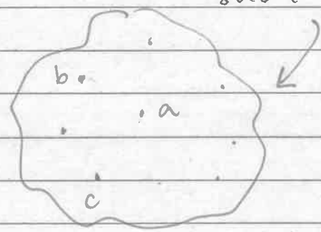
* Why equivalence relations?
→ Generalise the notion of equality, by giving a weaker version of equality.

* Equivalence classes



Let R be an equiv. relation on S.
Consider $a\in S$ (fixed), and also all pairs (a,b) ~~where $b\in S$~~ such that $(a,b)\in R$ and $b\in S$.

all things b ~~of the form~~ such that $(a,b)\in R$.



(piece of S)

$(a,b)\in R$, $(a,c)\in R$, $(a,a)\in R$ } we know this

$\Rightarrow$ $(b,a)\in R$, $(c,a)\in R$ } by symmetry.

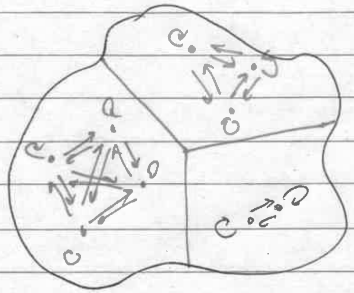
$(b,b), (c,c)\in R$ } by reflexivity

And $(b,c)\in R$ by transitivity
also (c,b) etc.

$\Rightarrow$ All the elements of this piece of S are related to all of the others in the same piece (*)

Nothing outside this piece can be related to anything in this piece (in any order).

As graphs, the pieces will look as follows:



No connections between pieces
Each piece fully connected internally.

These pieces are called <u>equivalence classes</u>.