Math 2301 28/07/2023

* Admin : - Gradescope coming soon!
  - HW 1 will release today on Wattle, due at 11:59 pm on Friday 04 Aug
  - Workshops start next week!
  - Worksheet available this weekend on Wattle. (Look at the problems in advance ~ 1 hour on your own.)
  - Office hrs starting next week Wed & Fridays.

* Last time : - Relations + properties
  - Directed graphs (to visualise)
  (Come back to graphs later in more detail.)

* Today : Equivalence relations.

Defn: A relation R on a set S is called an equivalence relation if it satisfies all of the following:

① Reflexivity → every (a,a) is in R

② Symmetry → $(a,b) \in R \Rightarrow (b,a) \in R$
i.e. if $(a,b) \in R$ then $(b,a) \in R$

③ Transitivity. → If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$.

* Examples and non-examples

- $S = \{1,2,3\}$

1. $S = \{1, 2, 3\}$

$R \subseteq S \times S$

$R = \{(a,b) \in S \times S \mid a = b\} = \{(1,1), (2,2), (3,3)\}$

Recall from Wed

$\rightarrow$ reflexivity, symmetry, transitivity.

so, it is an equivalence relation.

2. $S = \mathbb{Z}$ = set of integers.

$R = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} \mid a + b \text{ is even.}\}$

$= \{(0,0), (1,-1), (1,5), (2,24), (-3,15), \dots\}$

✓ reflexivity? yes, $(a,a) \in R$ b/c $a + a = 2a$ (even).

✓ symmetry? yes, if $a+b$ is even then $b+a$ is even.

✓ transitivity? suppose $(a,b) \in R$ and $(b,c) \in R$.

i.e. $a+b$ is even, and $b+c$ is even.

$\Rightarrow (a+b) + (b+c)$ is even.

$\Rightarrow (a+c) + 2b$ is even.

$\Rightarrow (a+c)$ is even.

$\Rightarrow (a,c) \in R$.

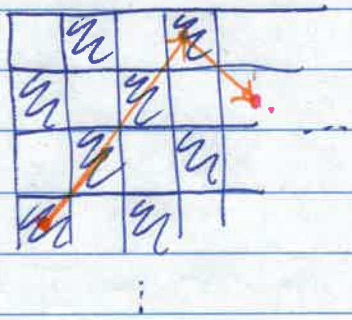
so, it is transitive.

so it is an equiv. relation.

3 $S = \mathbb{Z}$

$R = \{(a,b) \in \mathbb{Z}\times\mathbb{Z} \mid a+b \text{ is odd}\}$.

(Not an equivalence relation.)
→ Not reflexive!

4 

(8x8) chessboard.

S = set of squares on this chessboard.

$R = \{(a,b) \in S\times S \mid$ you can get to b from a by a sequence of bishop moves {(possibly, (no moves)}$\}$.

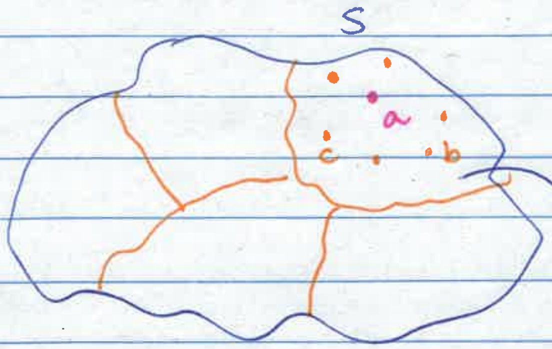
reflexivity? yes

symmetry? yes (move backwards)

transitivity? Let $(a,b) \in R$, $(b,c) \in R$.
go from $a \to b$, then $b \to c$,
gives you a sequence $a \to c$,
so $(a,c) \in R$.

* Why equivalence relations?
→ Generalise the notion of equality, by giving a weaker version of equality.

* Equivalence classes



Let R be an equiv. relation on S.

Consider $a \in S$ (fixed), and also all pairs (a,b) ~~where $b \in S$~~ such that $(a,b) \in R$ and $b \in S$.

all things b ~~of the form~~ such that $(a,b) \in R$.

(piece of S)

$(a,b) \in R$
$(a,c) \in R$
$(a,a) \in R$.
we know this

$\Rightarrow$ $(b,a) \in R$
$(c,a) \in R$
by symmetry.

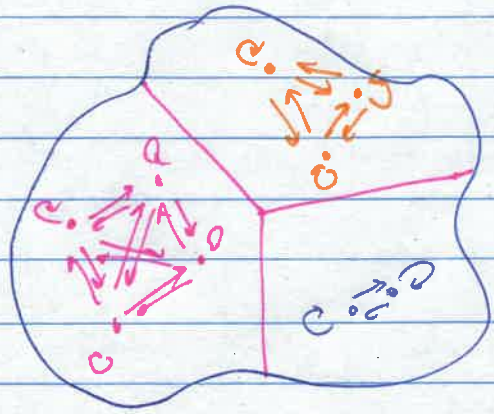
$(b,b), (c,c) \in R$ by reflexivity

And $(b,c) \in R$ by transitivity
also (c,b) etc.

$\Rightarrow$ All the elements of this piece of S are related to all of the others in the same piece *

Nothing outside this piece can be related to anything in this piece (in any order).

As graphs, the pieces will look as follows:



No connections between pieces
Each piece fully connected internally.

These pieces are called <u>equivalence classes</u>.