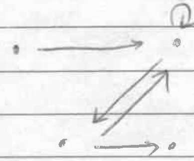
MATH 2301 04/08/2023

* Admin : - HW 1 due
  - RC 2 due by Sunday

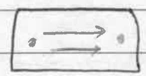
* Today : Directed graphs :

A graph (directed graph) consists of a set of "vertices" V and a relation $E \subseteq V \times V$ (edge relation)

We draw them:



- self loops are ok
- back & forth edges are ok
- multiple parallel edges not allowed



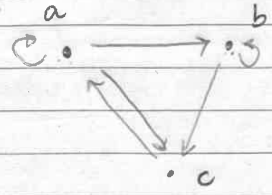
→ NOT ALLOWED*

* We may change this later ...

* Questions about networks:
  - Is there a path/route from A to B?
  - What is the shortest route?
  - How many routes?
  - Flow problems ...

↳ Main tool for us → adjacency matrix

Example



The set of vertices is $\{a, b, c\}$
Chose the ordering (a, b, c)

Adjacency matrix:

ROWS →

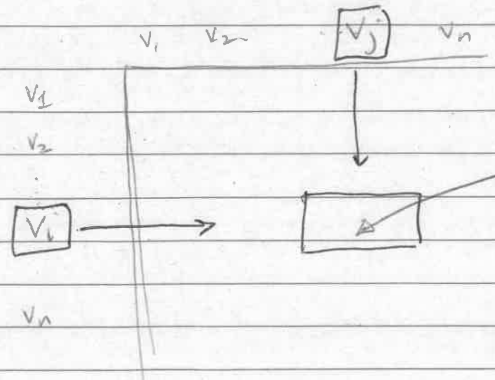
$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 1 & 1 \\ b & 0 & 1 & 1 \\ c & 1 & 0 & 0 \end{array}$$

COLUMNS

Defn : Let $G = (V, E)$ be a directed graph. Choose an ordering $(v_1, v_2, \dots, v_n)$ on V.

i.e. $V = \{v_1, \dots, v_n\}$ and $(v_1, v_2, \dots, v_n)$ is a chosen ordering.

The adjacency matrix for this ordering is an $(n \times n)$ matrix whose entry in row v_i (row i) and column v_j (column j) is 1 if $(v_i, v_j) \in E$ and 0 if $(v_i, v_j) \notin E$.

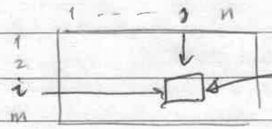


1 if $v_i \to v_j$ edge
0 if no edge $v_i \to v_j$

* Aside on matrix operations

Let A be an $m \times n$ matrix. → a grid of (typically) numbers.

(# rows) (# columns)



$(i,j)^{th}$ entry of A. called A_{ij}

** Addition / subtraction

If A and B are $(m \times n)$ matrices, their sum $(A \pm B)$ is the $(m \times n)$ matrix whose $(i,j)^{th}$ entry is

$A_{ij} \pm B_{ij}$

Example :
$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 6 \\ 0 & 5 & 8 \end{bmatrix}$$

** Scalar multiplication

If A is an $(m \times n)$ matrix and c is some number then

cA is the matrix whose $(i,j)^{th}$ entry is cA_{ij}

~~cA_{ij}~~ Example :
$$3 \cdot \begin{bmatrix} 1 & 0 & 2 \\ -1 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 6 \\ -3 & 15 & 3 \end{bmatrix}$$

** Matrix product :

Let A be an $m \times k$ matrix
Let B be a $k \times n$ matrix

We have a product matrix $A \cdot B$ or AB → will be $(m \times n)$
constructed as follows:

Example

$$\underset{(2\times 3)}{A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix}} \qquad \underset{(3\times 2)}{B = \begin{bmatrix} 4 & 0 \\ -1 & 5 \\ 2 & 1 \end{bmatrix}}$$

The $(i,j)^{th}$ entry of (AB) is constructed as the dot product of the i^{th} row of A with the j^{th} column of B.

$$AB = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} \boxed{13} & 8 \\ 3 & 7 \end{bmatrix} \end{array}$$

$\boxed{13} \rightarrow (2,1,3)\cdot(4,-1,2)$

$= 8 + (-1) + 6$

$= 13$