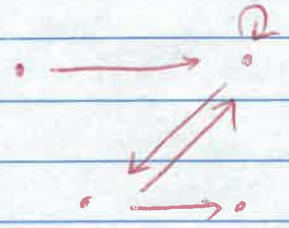
MATH 2301 04/08/2023

* **Admin**: - HW 1 due
  - RC 2 due by Sunday

* **Today**: Directed graphs:

A graph (directed graph) consists of a set of "vertices" V and a relation $E \subseteq V \times V$
$\downarrow$
(edge relation)

We draw them:



- self loops are ok
- back & forth edges are ok

- multiple parallel edges not allowed.

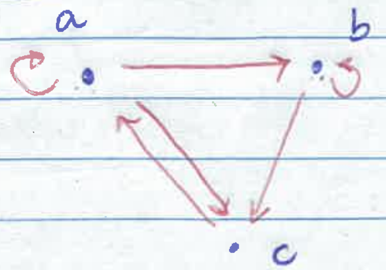
 $\longrightarrow$ NOT ALLOWED*

* We may change this later...

* **Questions** about networks:
  - Is there a path/route from A to B?
  - What is the shortest route?
  - How many routes?
  - Flow problems...

$\hookrightarrow$ Main tool for us $\longrightarrow$ adjacency matrix.

Example



The set of vertices is $\{a, b, c\}$
Choose the ordering (a, b, c)

Adjacency matrix:

Rows

$$
\begin{array}{c|ccc|}
 & a & b & c \\
\hline
a & 1 & 1 & 1 \\
b & 0 & 1 & 1 \\
c & 1 & 0 & 0 \\
\hline
\end{array}
$$

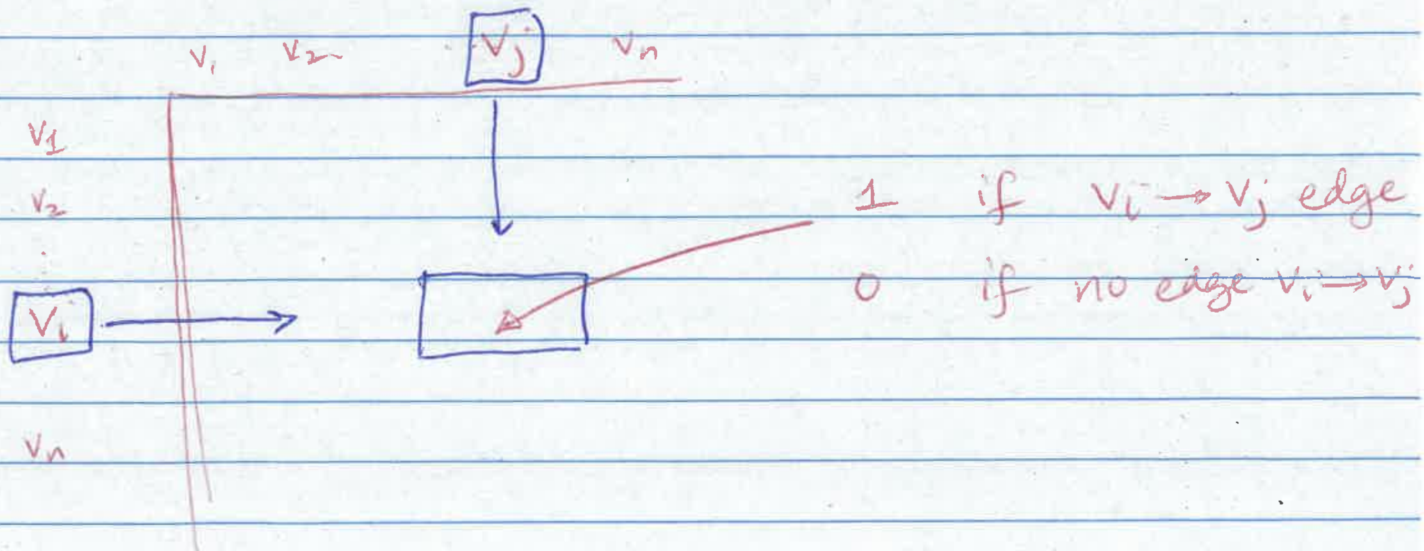
COLUMNS

Defn: Let $G = (V, E)$ be a directed graph.
Choose an ordering $(v_1, v_2, \cdots, v_n)$ on V.

i.e. $V = \{v_1, \cdots, v_n\}$ and $(v_1, v_2, \cdots, v_n)$ is a chosen ordering.

The adjacency matrix for this ordering is an $(n \times n)$ matrix whose entry in row v_i (row i) and column v_j (column j)

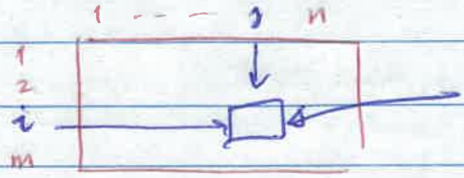
is 1 if $(v_i, v_j) \in E$ and 0 if $(v_i, v_j) \notin E$.



* Aside on matrix operations

Let A be an $m \times n$ matrix. → a grid of (typically) numbers.

m (# rows), n (# columns)



→ $(i,j)^{th}$ entry of A. called A_{ij}

** Addition / subtraction

If A and B are $(m \times n)$ matrices, their sum $(A \pm B)$ is the $(m \times n)$ matrix whose $(i,j)^{th}$ entry is

$$A_{ij} \pm B_{ij}$$

Example :
$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 6 \\ 0 & 5 & 8 \end{bmatrix}$$

** Scalar multiplication

If A is an $(m \times n)$ matrix and c is some number then

cA is the matrix whose $(i,j)^{th}$ entry is cA_{ij}

Example :
$$3 \cdot \begin{bmatrix} 1 & 0 & 2 \\ -1 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 6 \\ -3 & 15 & 3 \end{bmatrix}$$

** Matrix product :

Let A be an $m \times k$ matrix.
Let B be a $k \times n$ matrix.

will be $(m \times n)$

We have a product matrix $A \cdot B$ or AB constructed as follows:

Example

(2×3) $\quad$ (3×2)

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 0 \\ -1 & 5 \\ 2 & 1 \end{bmatrix}$$

The $(i,j)^{th}$ entry of (AB) is constructed as the dot product of the i^{th} row of A with the j^{th} column of B.

$$AB = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} \boxed{13} & 8 \\ 3 & 7 \end{bmatrix} \end{array}$$

$$(2,1,3)\cdot(4,-1,2)$$
$$= 8 + (-1) + 6$$
$$= 13$$