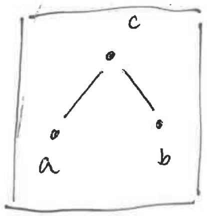
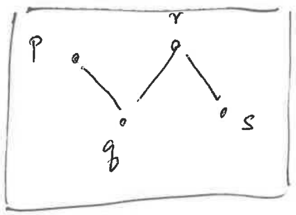
MATH 2301 18/08/2023 ①

* Partially ordered sets

Hasse diagrams

- Maximal, minimal, maximum, minimum elements





c, p, r are maximal.
a, b, q, s are minimal.
c is a maximum.
no minimum.

Note : - a and b are at "bottom level", but not less than or equal to everything

- c ~~each~~ is bigger than or equal to everything else.

- p, r are at the "top level", but not ~~that~~ bigger than or equal to everything.

*** Def** : Let $(P, \preceq)$ be a poset.

(1) An element $x \in P$ is maximal if there is no $y \neq x$ such that $x \preceq y$.

(2) An element $x \in P$ is minimal if there is no $y \neq x$ such that $y \preceq x$.

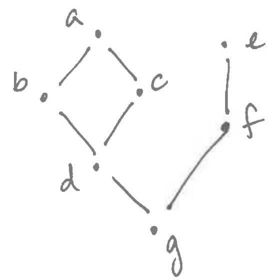
(3) An element $x \in P$ is maximum if for all $y \in P$, we have $y \preceq x$.

(4) An element $x \in P$ is minimum if for all $y \in P$, we have $x \preceq y$.

②

* Topological sortings

- Let $(P, \preceq)$ be a (finite) poset.



$P = \{a, b, c, d, e, f, g\}$.

Want : An ordered tuple of the elements of P, satisfying a special property: whenever $x \preceq y$ in P, I want x to appear before y in the ordered tuple.

→ this is a topological sorting

Example ① (g, f, d, e, c, b, a)

[Blue arrows are edges in Hasse diagram.]

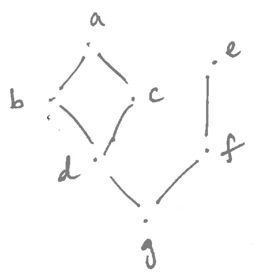
② (g, d, f, c, e, b, a)

Note : There can be many topological sortings of a given poset.

Def : An ordering $(v_1, v_2, \dots, v_n)$ of the elements of a poset $(P, \preceq)$ is called a topological sorting if : whenever $v_i \preceq v_j$, we have $i \leq j$.
That is, v_i appears before v_j in the list.

* Why topological sortings?

<u>One answer:</u> adjacency matrix



<u>Option 1:</u> use (a, b, c, d, e, f, g) as ordering for A

<u>Option 2:</u> use a topological sorting, e.g. (g, f, d, e, c, b, a) as ordering for A.

<u>Option 1</u>

$$\begin{array}{c|ccccccc} & a & b & c & d & e & f & g \\ \hline a & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ c & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ d & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ e & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ f & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ g & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}$$

<u>Option 2</u>

$$\begin{array}{c|ccccccc} & g & f & d & e & c & b & a \\ \hline g & 1 & & & & & & \\ f & & 1 & & & & & \\ d & & & \ddots & & & & \\ e & & & & \ddots & & & \\ c & & & & & \ddots & & \\ b & & & & & & 1 & \\ a & & & & & & & \end{array}$$

* <u>Upshot:</u> In a topological sorting, the adjacency matrix has zeroes below the diagonal [it is an upper-Δ matrix]

↳ <u>return on Monday</u>.