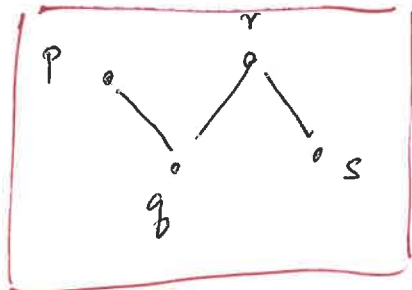
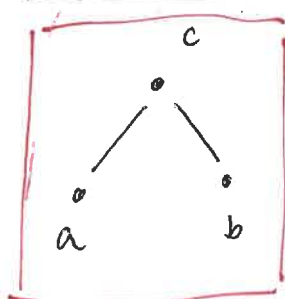


\* Partially ordered sets

Hasse diagrams

- Maximal, minimal, maximum, minimum elements



*c, p, r are maximal.  
a, b, q, s are minimal.  
c is a maximum.  
no minimum.*

- Note:
- a and b are at "bottom level", but not less than or equal to everything
  - c ~~is~~ is bigger than or equal to everything else.
  - p, r are at the "top level", but not ~~is~~ bigger than or equal to everything.

\* Def: Let  $(P, \leq)$  be a poset.

(1) An element  $x \in P$  is maximal if there is no  $y \neq x$  such that  $x \leq y$ .

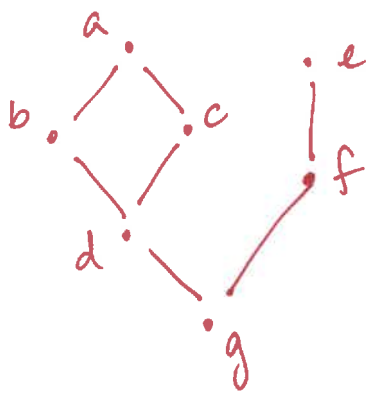
(2) An element  $x \in P$  is minimal if there is no  $y \neq x$  such that  $y \leq x$ .

(3) An element  $x \in P$  is maximum if for all  $y \in P$ , we have  $y \leq x$ .

(4) An element  $x \in P$  is minimum if for all  $y \in P$ , we have  $x \leq y$ .

# \* Topological sortings

Let  $(P, \leq)$  be a (finite) poset.



$$P = \{a, b, c, d, e, f, g\}$$

Want: An ordered tuple of the elements of  $P$ , satisfying a special property: whenever  $x \leq y$  in  $P$ , I want  $x$  to appear before  $y$  in the ordered tuple.

→ this is a topological sorting

Example ①  $(g, f, d, e, c, b, a)$

[Blue arrows are edges in Hasse diagram.]

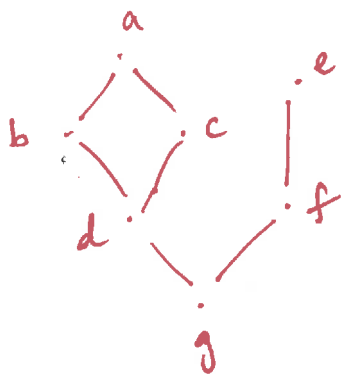
②  $(g, d, f, c, e, b, a)$

Note: There can be many topological sortings of a given poset.

Def: An ordering  $(v_1, v_2, \dots, v_n)$  of the elements of a poset  $(P, \leq)$  is called a topological sorting if: whenever  $v_i \leq v_j$ , we have  $i \leq j$ . That is,  $v_i$  appears before  $v_j$  in the list.

\* Why topological sortings?

One answer: adjacency matrix



Option 1: use (a, b, c, d, e, f, g) as ordering for A

Option 2: use a topological sorting, e.g. (g, f, d, e, c, b, a) as ordering for A.

Option 1

	a	b	c	d	e	f	g
a	1	0	0	0	0	0	0
b	1	1	0	0	0	0	0
c	1	0	1	0	0	0	0
d	1	1	1	1	0	0	0
e	0	0	0	0	1	0	0
f	0	0	0	0	1	1	0
g	1	1	1	1	1	1	1

Option 2

	g	f	d	e	c	b	a
g	1						
f		1					
d			1				
e				1			
c					1		
b						1	
a							1

\* Upshot: In a topological sorting, the adjacency matrix has zeros below the diagonal [it is an upper-Δ matrix]

↳ return on Monday.