MATH 2301 25/09/2023

* Last time: A word w matches a regex r if $w \in L(r)$.

Example

Given $L = \{w \in \Sigma^* \mid w$ contains an even number of 1s$\}$

find a regex r such that $L(r) = L$.

Attempts from Friday:

$r = (11)^*$ ← not good; misses e.g. 101

$r = (10^*1)^*$ ← not good; misses 0101

$r = (\underbrace{0^*10^*10^*}_{}) ^*$ ← not good; misses 000

↳ matches anything with exactly 2 1's

$r = (0^*10^*10^*)^* \mid 0^*$ [convince yourself that this one works!]

* Notation: If $a \in \Sigma$, and $k \in \mathbb{N}$, then a^k denotes a k-fold concatenation of a.

$\underbrace{aa\cdots a}_{k \text{ times}}$

Technically not a regex, but often used as a shorthand.

(!) (**) Try not to use this if asked to produce a regex!!

** Questions:

From Friday, we have a procedure that takes in regexes and tells you the language of the regex.

E.g. $L(0^*) = (L(0))^* = (\{0\})^*$

(*) Can you go backwards? Given a description of a language L, can we always produce a regex r such that $L = L(r)$?

We'll investigate...

(*) If $L = L(r)$ for some r, then is r the only regex with this property?

Previous example: $r = (0^*10^*10^*)^* \mid 0^*$

Other options: $0^* \mid (0^*10^*10^*)^*$,

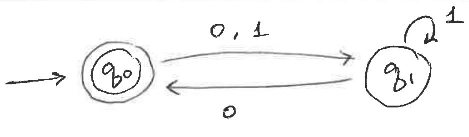
$0^* \mid (0^*10^*10^*)^* \mid 00$

$\boxed{\text{NO}}$. There can be many regexes that produce the same language.

* Automata: If a human/machine tries to check whether some string satisfies some pattern, it is more natural to read the string letter by letter, to check the pattern.

* Deterministic Finite Automata (DFAs)

Example (state diagram)



E.g. $w = 1011$

$q_0 \xrightarrow{1} q_1$

$q_1 \xrightarrow{0} q_0$

$q_0 \xrightarrow{1} q_1$

$q_1 \xrightarrow{1} q_1$

** Def: A DFA consists of the following:

(1) An alphabet Σ

(2) A finite set of "states" or vertices Q.

(3) A start state $q_0 \in Q$ (denoted by dangling incoming arrow $\to q_0$)

(4) A set $A \subseteq Q$ of "accepting/accept" states (denoted by a double circle q_1)

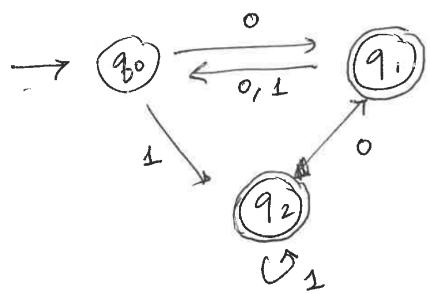
(5) A transition function

$\delta : Q \times \Sigma \longrightarrow Q$. (Move from q to q' if you see the letter a)

$(q, a) \longmapsto q'$

* Note: Given a DFA, you can draw its state diagram as in the first example.

E.g.



$\Sigma = \{0, 1\}$

$Q = \{q_0, q_1, q_2\}$

$A = \{q_1, q_2\}$

start state is q_0

δ ?

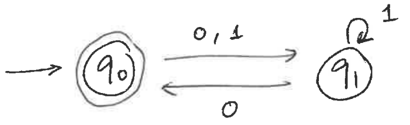
Q	Σ	output of δ
q_0	0	q_1
q_0	1	q_2
q_1	0	q_0
q_1	1	q_0
q_2	0	q_1
q_2	1	q_2

[Follow the arrows in the state diagram]

Note: A label of $0, 1$ is equivalent to writing Σ as a shorthand, if $\Sigma = \{0, 1\}$

More generally, if $\Sigma = \{a_1, a_2, \dots, a_n\}$ then you may use Σ to denote the label $a_1, a_2, \dots, a_n$.

E.g.



An example calculation

$w = \underline{1}011$

Read w from left to right, letter by letter, starting at q_0, and applying δ as we go

$q_0 \xrightarrow{1} q_1$ is equivalent to saying $\delta(q_0, 1) = q_1$

$q_1 \xrightarrow{0} q_0$; i.e $\delta(q_1, 0) = q_0$

$q_0 \xrightarrow{1} q_1$

$q_1 \xrightarrow{1} \boxed{q_1}$ ← last state we reach at the end of the string

If this state is in A, return $\boxed{\text{ACCEPT}}$

If not, return. $\boxed{\text{REJECT}}$

Note: If M is a DFA, we say that L(M) is the set of all strings accepted by M.