MATH 2301 13/10/2023

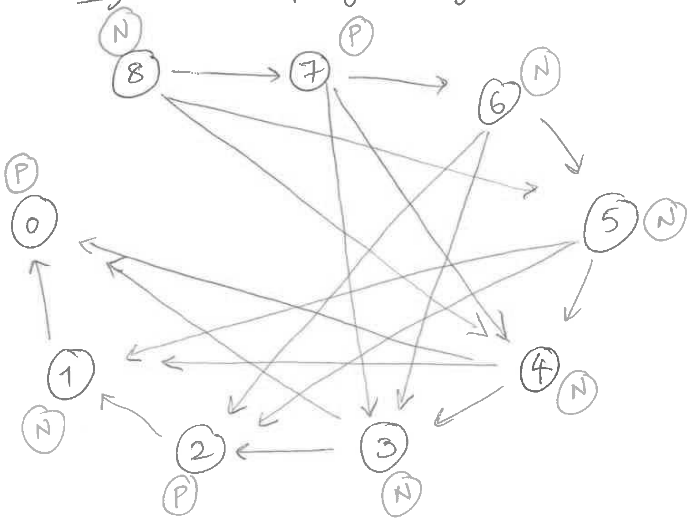
* Combinatorial games

E.g. Subtraction game $S=\{1,3,4\}$

Game state is some $n \geq 0$

A game move = subtract a single element of S from n.

E.g. $n=8$; game graph:



P $\leftrightarrow$ "previous player wins"

N $\leftrightarrow$ "next player wins"

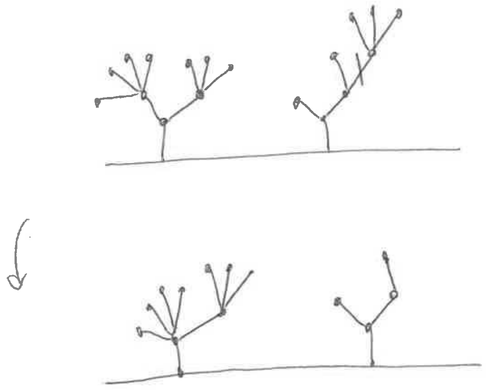
(wins means "has a winning strategy")

- Every state w/out outgoing arrows is labelled "P".
- Every state that points to a P-state by an arrow is labelled "N".
- Every state that only points to "N" states by arrows is labelled as "P".
- Work backwards along the game graph.
- An optimal move from an "N"-state consists of moving to a state labelled "P".

* Some examples of games.

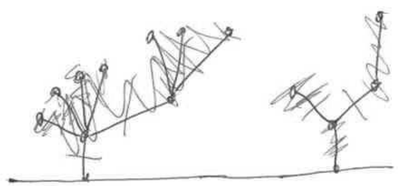
** Hackenbush.

Game states look like:



A move consists of chopping off a segment.

Anything that is not connected to the ground dies (is deleted).

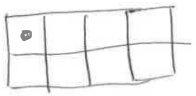


One could draw the game graph, and label it by N & P.

But it would be very long calculation.

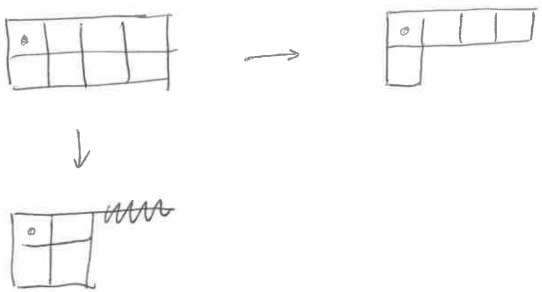
** Chomp

Starting game state: $(m \times n)$ bar of chocolate

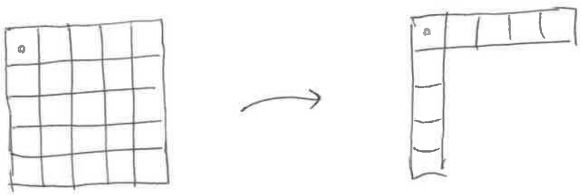


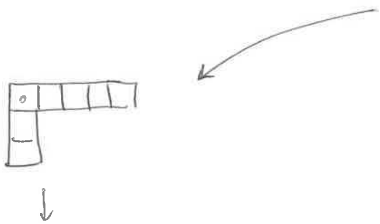
Top left square is poisoned and inedible.

A move consists of choosing a non-poisoned square and eating everything in the bottom-right quadrant of this.

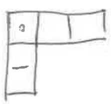


Special case: start with a square bar.





This is a P-position; because anything that the next player does, the previous player can mirror. (on the other leg)



$\Rightarrow$ Any $(m \times m)$ square (except $m=1$) is an N-position.