MATH 2301

13/10/2023

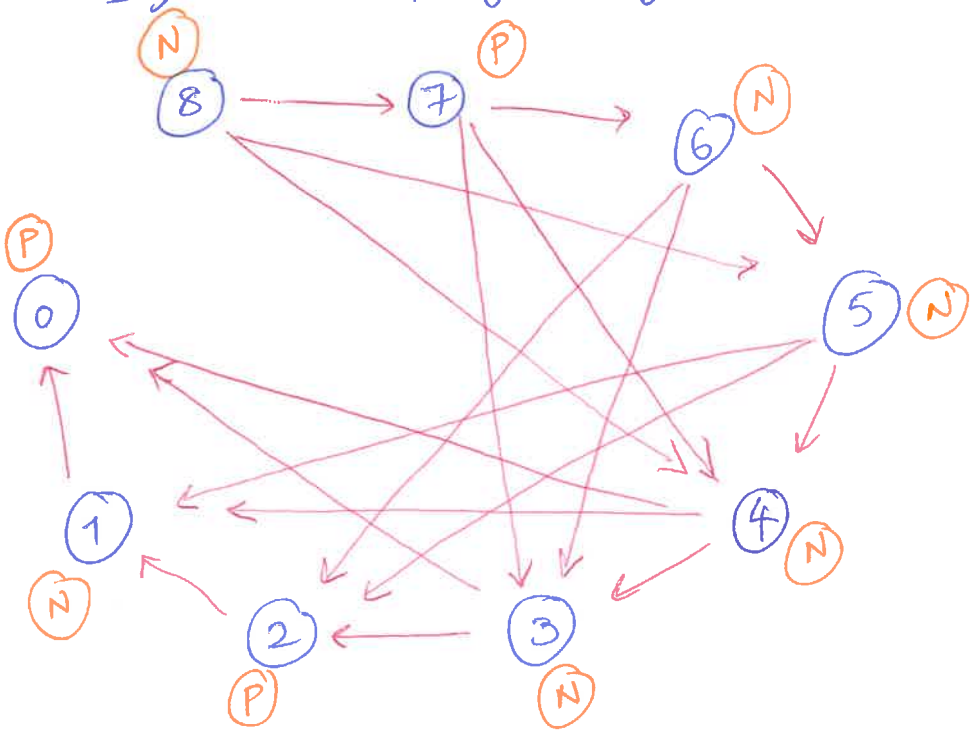
* Combinatorial games

E.g. Subtraction game $S = \{1, 3, 4\}$

Game state is some $n \geq 0$

A game move = subtract a single element of S from n.

E.g. $n = 8$; game graph:



P ↔ "previous player wins"

N ↔ "next player wins".

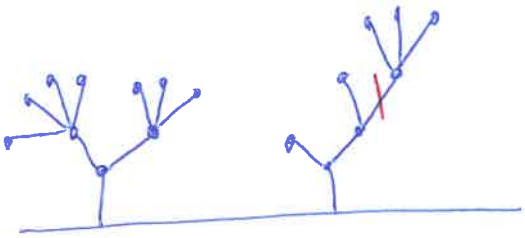
(wins means "has a winning strategy")

- Every state w/out outgoing arrows is labelled "P".
- Every state that points to a P-state by an arrow is labelled "N".
- Every state that only points to "N" states by arrows is labelled as "P".
- Work backwards along the game graph.
- An optimal move from an "N"-state consists of moving to a state labelled "P".

* Some examples of games:

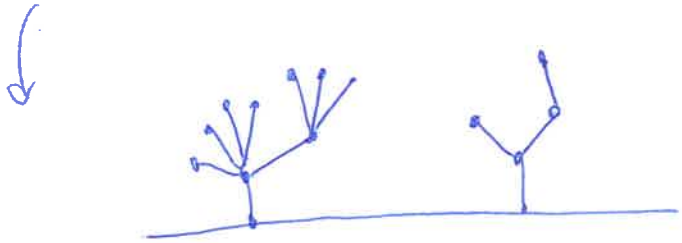
** Hackenbush.

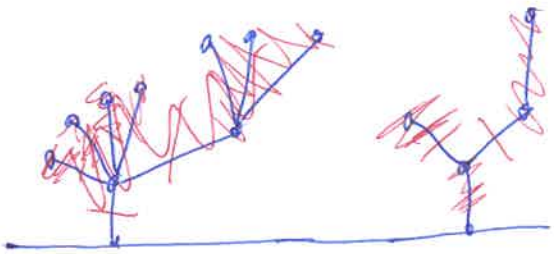
Game states look like:



A move consists of chopping off a segment.

Anything that is not connected to the ground dies (is deleted).

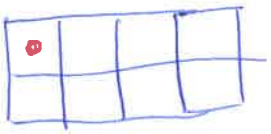




One could draw the game graph, and label it by N & P.
But it would be very long calculation.

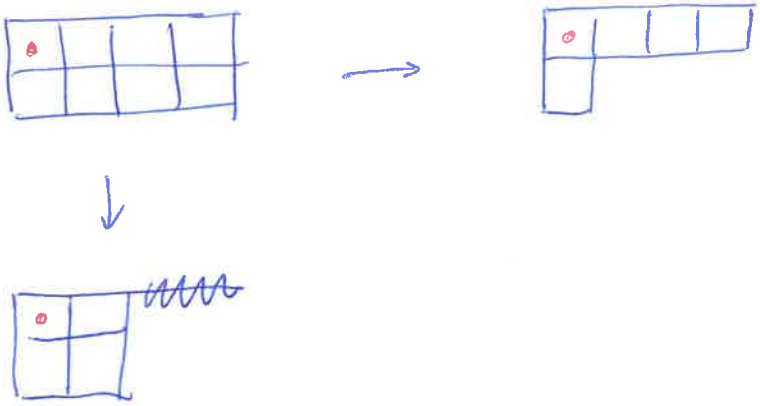
** Chomp

Starting game state: (m x n) bar of chocolate

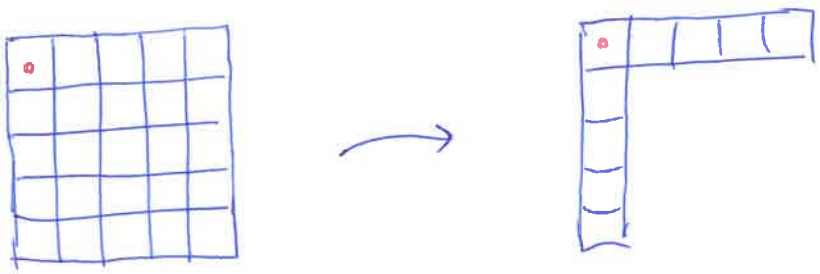


Top left square is poisoned and inedible.

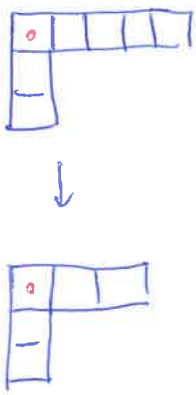
A move consists of choosing a non-poisoned square and eating everything in the bottom-right quadrant of this.



Special case: start with a square bar.



This is a P-position; because anything that the next player does, the previous player can mirror. (on the other leg)



⇒ Any (m x m) square (except m=1) is an N-position.