MATH 2301

20/10/2023

* **Admin**: ① Final exam prep problems → Wattle by Monday.

② EAPs handled by exam office directly, so wait for them to be in touch about any special arrangements.

③ Final exam prep extra office hrs → see Wattle by this afternoon.

* **Last time**:

① If $(m_1, \dots, m_k)$ is a nim position with $m_1 \oplus \dots \oplus m_k > 0$, then there is always at least one move $m_i \to m_i'$ which makes the nim-sum zero.

(Make a move in any pile which has a 1 in the binary representation at the same column ~~xxxxxx~~ as the leftmost 1 in the binary repr. of the nim-sum $(m_1 \oplus \dots \oplus m_k)$)

② If $(m_1, \dots m_k)$ has $m_1 \oplus \dots \oplus m_k = 0$ then any move makes the nim-sum > 0.

(Discussed last time)

* Thm $(m_1, \dots, m_k)$ is an N-position in nim if and only if $m_1 \oplus \cdots \oplus m_k > 0$.
It is a P-position if and only if $m_1 \oplus \cdots \oplus m_k = 0$.

Explanation: Winning strategy for a position with positive sum consists of taking it to a zero-nim sum.

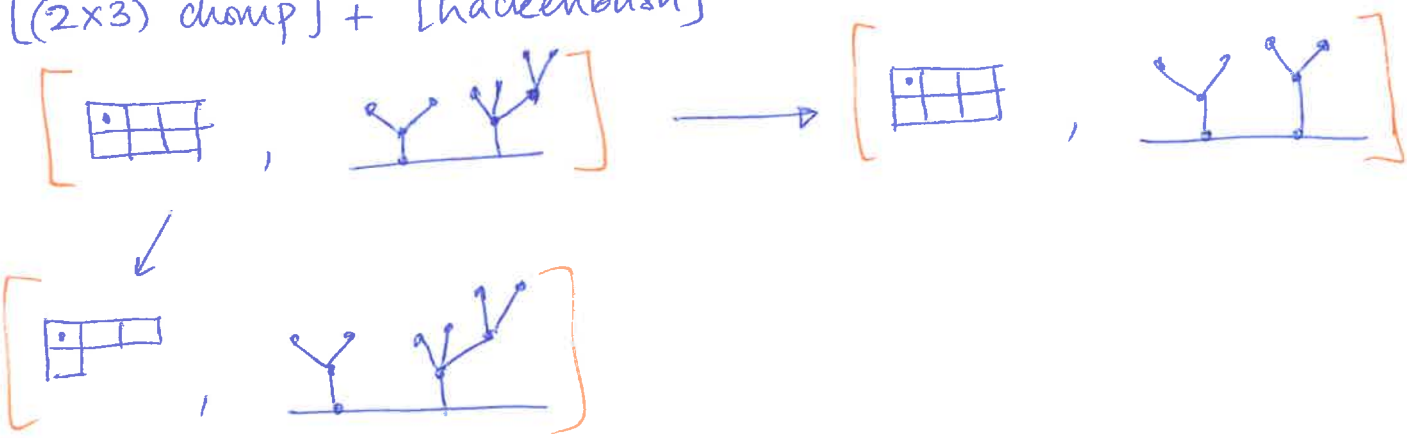
(More formally: can use induction / structural induction on game graph.)

* Today: Sums of games / game positions.
(Often, "game" means game state or game position)

Def: If G and H are game states, then $G+H$ is the game state for a new game, where an allowed move is simply an allowed move in one of the two games.

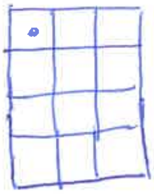
$G+H \longrightarrow G'+H$ OR $G+H \longrightarrow G+H'$ (not both)

Example
[(2×3) chomp] + [hackenbush]



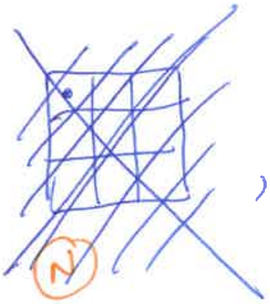
Rmk: A nim game $(m_1, \ldots, m_k)$ is the sum of k single-pile nim games with sizes $m_1, m_2, \ldots, m_k$.

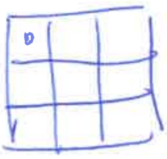
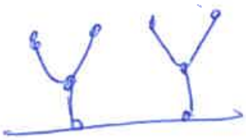
Examples:

①  , $(1, 3)$ (chomp + nim)

N → (Wikipedia) N (nim-sum is 2)

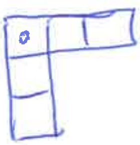
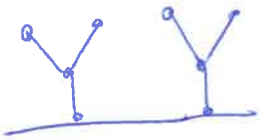
We're not sure if the full game is N or P.

②  ,  (chomp + hackenbush)

N P

The full game is an N-game. Why?
The best move for P1 is to go to:

( , )

P P

If P2 makes a move in chomp, mirror it
If P2 makes a move in hackenbush, mirror it.

* **Def**: The outcome of a game G is either N or P depending on whether the next player has a winning strategy. We say that G_1 & G_2 have the same outcome if they are both N or both P.

* **Def**: We say that two games G_1 & G_2 are equivalent if for any game H, the games $G_1 + H$ and $G_2 + H$ have the same outcome.

Rmks:

1. If G_1 and G_2 are equivalent according to this definition, then G_1 and G_2 have the same outcome.
   (You can add the empty game ϕ to both sides, where ϕ is the game where there are no allowed moves, and the game state is ϕ.)

2. Even if G_1 & G_2 have the same outcome, they may not be equivalent.
   [Do examples to understand this.]

3. This ~~eq~~ relation defined above is an equivalence relation.
   [Exercise!]