Symplectic Reflection Algebras: the KZ functor and quiver varieties — Lecture 1

Example: Let $X = T^*\mathcal{F}l = \{(A,F) \in \mathrm{Mat}_n(\mathbb{C}) \times \mathcal{F}l \mid AF_i \subseteq F_{i-1}\}$.

$$\begin{tikzcd} & X \arrow[dl, "\mu"'] \arrow[dr, "\pi"] & \\ \mathcal{N} & & \mathcal{F}l \end{tikzcd}$$

For $n=2$, check this is the Kleinian singularity and its resolution is $\mathbb{C}^2/\mu_2$.

Definition: A normal affine variety Y has <u>symplectic singularities</u> if:

(i) there is $\varphi \in \Omega^2_{Sm(Y)}$ which is closed and non-degenerate (i.e. $Sm(Y)$ is symplectic)

(ii) for some (any) resolution $f: X \to Y$, $f^*\varphi$ extends from $f^{-1}(Sm(Y))$ to a closed 2-form on all of X.

Y has <u>conical symplectic singularities</u> if, additionally:

(iii) there is a $\mathbb{C}^\times$-action on Y with $\lambda \cdot \varphi = \lambda^i \varphi$ for $i > 0$ and $\mathbb{C}[Y] = \bigoplus_{j \geq 0} \mathbb{C}[Y]_j$, $\mathbb{C}[Y]_0 = \mathbb{C}$.

Recall $\mathfrak{g}^* \simeq \mathfrak{g}$ (as G-modules), and on $\mathfrak{g}^*$ there is a Poisson structure via $\{A,B\} = [A,B]$. $A, B \in \mathfrak{g} \subseteq \mathcal{O}(\mathfrak{g}^*)$ restricts to $\mathcal{N}$. $T^*\mathcal{F}l$ is the cotangent bundle to a manifold and so admits a symplectic structure.

Example: If $X_0 = V$ a vector space, $T^*X_0 = V \oplus V^*$ with $\omega(v,v') = 0 = \omega(f,f')$, $\omega(v,f) = f(v)$ for $v,v' \in V$, $f,f' \in V^*$ being the symplectic structure from the Liouville form.

Definition: Suppose Y has symplectic singularities. Then $f: X \to Y$ is a <u>symplectic resolution</u> if f is a resolution and the form on $f^{-1}(Sm(Y))$ extends to a non-degenerate closed 2-form on X.

Examples: (i) The Springer resolution $\mu: T^*\mathcal{F}l \to \mathcal{N}$ is a symplectic resolution.
(ii) Kleinian singularities: G acts on V, then $(T^*V)/G$ has symplectic singularities but not symplectic resolutions.

Let G be a reductive group acting on an affine variety Z. Then $Z/\!\!/G = \mathrm{Spec}(\mathbb{C}[Z]^G)$, and the surjective morphism $Z \twoheadrightarrow Z/\!\!/G$ corresponds to $\mathbb{C}[Z]^G \hookrightarrow \mathbb{C}[Z]$, where each orbit contains a unique closed orbit.

Institute of Science and Technology Austria | Am Campus 1 | 3400 Klosterneuburg | Austria
Phone +43 (0) 2243/9000 | Fax +43 (0) 2243/9000 - 2000 | office@ist.ac.at | www.ist.ac.at

Example: Let $\mathbb{C}^\times = G$, $Z = \mathbb{C}^n$, $\lambda \cdot z = \lambda^{-1} z$. Then $\mathbb{C}^n /\!\!/ \mathbb{C}^\times = pt$.

Let $\theta : G \to \mathbb{C}^\times$ be a character. Set:

$$Z^{\theta-ss} := \{ z \in Z \mid \exists f \in \mathbb{C}[Z]^{G, n\theta}, n > 0, f(z) \neq 0 \} \subseteq_o Z, \text{ where}$$

$$\mathbb{C}[Z]^{G,n\theta} := \{ f \in \mathbb{C}[Z] \mid f(g^{-1}z) = \theta(g)^n f(z) \}$$

Then Z is the union for principal open subvarieties $f \in \mathbb{C}[Z]^{G, n\theta}$

$$\begin{array}{c} \text{Set } Z/\!\!/_\theta G := Z^{\theta-ss}/\!\!/G \longleftrightarrow \mathrm{Proj}\left(\bigoplus_{n \geq 0} \mathbb{C}[Z]^{G, n\theta}\right) \\ \downarrow \\ Z/\!\!/G = (Z/\!\!/_0 G) \end{array}$$

Example: $\theta(t) = t^m$, $m > 0$: $Z^{\theta-ss} = \mathbb{C}^n \setminus \{0\}$ and $Z/\!\!/_\theta G = \mathbb{P}^{n-1}$. If $m < 0$, $Z^{\theta-ss} = \emptyset$.

Let the G-action on X be a symplectic-preserving form. This induces a map $\xi : \mathfrak{g} \to \mathrm{Vect}$
$\xi(A) = \xi_A$. Then

$$\begin{array}{ccc} \mathbb{C}[X] & \longrightarrow & \mathrm{Vect}(X) \\ \Psi & & \\ f & \longmapsto & \mathrm{Ham}(f) \; [\text{defined by } \omega(\mathrm{Ham}(f), -) := -df]. \end{array}$$

The action is <u>hamiltonian</u> if the following diagram can be completed:

$$\begin{tikzcd} \mathfrak{g} \arrow[r, "\xi"] \arrow[d, dashed, "\mu^\#"'] & \mathrm{Vect}(X) \\ \mathbb{C}[X] \arrow[ur] & \end{tikzcd}$$

$X \xrightarrow{\mu} \mathfrak{g}^*$ the moment map.

Exercise: The G-action on X_0 is hamiltonian with $\mu^\#(A) = \xi_A \in TX_0 \subseteq \mathbb{C}[T^*X_0]$.

Given $\lambda \in (\mathfrak{g}^*)^G$, define $X/\!\!/\!\!/_\lambda G := \mu^{-1}(\lambda)/\!\!/G$. This has dimension $\dim X - 2\dim G$ generically, and is symplectic if G acts freely.

Let $Q = (Q_0, Q_1, t, h)$ be a quiver. A <u>framed representation</u> has, for $k \in Q_0$, a map of finite-dimensional vector spaces $i_k : V_k \to W_k$, $a \in Q_1$: $V_{t(a)} \xrightarrow{z_a} V_{h(a)}$:
$\underline{v} = (\dim V_k)_{k \in Q_0}$, $\underline{w} = (\dim W_k)_{k \in Q_0}$. Then:

$$R := \mathrm{Rep}(Q, \underline{v}, \underline{w}) = \bigoplus_{k \in Q_0} \mathrm{Hom}(V_k, W_k) \oplus \bigoplus_{a \in Q_1} \mathrm{Hom}(V_{t(a)}, V_{h(a)})$$

and there is a $G(\underline{v}) := \prod_{k \in Q_0} GL(V_k)$-action on R.

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For geometric invariant theory, $\theta \in \mathbb{Z}^{Q_0}$, $G(\underline{v}) \longrightarrow \mathbb{C}^\times$ via $(g_k) \longmapsto \prod_{k\in Q_0} \det(g_k)^{\theta_k}$.

$$T^*R = \bigoplus \left(\mathrm{Hom}(V_k, W_k) \oplus \mathrm{Hom}(W_k, V_k)\right) \oplus \qquad (\ni (i_k, j_k))$$
$$\bigoplus \left(\mathrm{Hom}(V_{t(a)}, V_{h(a)}) \oplus \mathrm{Hom}(V_{h(a)}, V_{t(a)})\right) \quad (\ni (x_a, x_{a^*})).$$

The moment map $\mu : T^*R \longrightarrow \mathfrak{g}^* = \mathrm{Lie}(G(\underline{v})^*) \simeq \bigoplus_{k\in Q_0} \mathrm{Hom}(V_k, V_k)$

is given by $(x_a, x_{a^*}, i_k, j_k) \longmapsto \sum_{a\in Q_1} (x_a x_{a^*} - x_{a^*} x_a) - \sum_{k\in Q_0} i_k j_k$.

<u>Definition</u>: A <u>Nakajima quiver variety</u> is of the form, for $\theta \in \mathbb{Z}^{Q_0}$:

$$M^\theta(\underline{v}, \underline{w}) = T^*R /\!\!/\!\!/_0^{\theta}\, G(\underline{v})$$

<u>Proposition</u>: There is $\pi : M^\theta(\underline{v}, \underline{w}) \longrightarrow M^0(\underline{v}, \underline{w})$, which has symplectic singularities.

Lecture 2

Let Z have conical symplectic singularities, $\mathbb{C}[Z] = \bigoplus_{j\geq 0} \mathbb{C}[Z]_j$

<u>Definition</u>: A <u>quantisation</u> of Z is an $\mathbb{N}$-filtered algebra $\mathcal{U}$ s.t. $\mathrm{gr}\,\mathcal{U} \simeq \mathbb{C}[Z]$ as graded Poisson algebras

In this situation, there is a filtration $\dots \subseteq F^j\mathcal{U} \subseteq F^{j+1}\mathcal{U} \subseteq \dots$ with $\bigcup_{j\in\mathbb{N}} F^j\mathcal{U} = \mathcal{U}$: $\mathrm{gr}\,\mathcal{U} = \bigoplus_{j\in\mathbb{N}} \frac{F^j\mathcal{U}}{F^{j-1}\mathcal{U}}$ and the Poisson structure is given by:

$$\{x + F^{j-1}\mathcal{U}, y + F^{k-1}\mathcal{U}\} = [x,y] + F^{j+k-2}.$$

<u>Exercises</u>: (i) $\mathcal{U} = \mathcal{U}(\mathfrak{g})$ quantises $S(\mathfrak{g}) = \mathcal{O}(\mathfrak{g}^*)$

(ii) $\mathcal{U} = \mathcal{U}(\mathfrak{g})_\lambda := \frac{\mathcal{U}(\mathfrak{g})}{m_\lambda \mathcal{U}(\mathfrak{g})}$, $m_\lambda \trianglelefteq Z(\mathcal{U}(\mathfrak{g}))$ $(\lambda \in \mathfrak{h}^*/W)$ quantises $\mathcal{O}(\mathcal{N})$

(iii) $\mathcal{U} = D(V)$ (global differential operators on V) quantises $\mathbb{C}[T^*V]$

Let X be a $\mathbb{C}^\times$-Poisson variety, gluing affine pieces. However:
(i) $U \subseteq X$ open need not be $\mathbb{C}^\times$-stable
(ii) $U \subseteq X$ open $\mathbb{C}^\times$-stable need not be positively graded.

fix this, we instead:
(i) consider the ~~canonical~~ conical topology (Zariski open and $\mathbb{C}^\times$-stable subsets)
(ii) use $\mathbb{Z}$-filtrations instead of $\mathbb{N}$-filtrations (complete and separated).

Definition: A quantisation of X is a sheaf $\mathcal{E}$ of $\mathbb{Z}$-filtered algebras s.t. $\operatorname{gr} \mathcal{E} \xrightarrow{\sim} \mathcal{O}_X$ is a Poisson isomorphism of graded sheaves of algebras.

Example: Let $\mathbb{C}[T^*V] = \mathbb{C}[x_1, \dots x_n, y_1, \dots y_n]$, D_V a sheaf on V. We define a new product on $\mathbb{C}[T^*V]$ by:

$(\mathbb{C}[T^*V], *)$: $f * g = (\text{mult} \circ \exp(v))(f \otimes g)$, where

$v = \sum_{i=1}^{n} \partial_{y_i} \otimes \partial_{x_i}$ Moyal product $[\exp v = \sum_{n \geq 0} \frac{1}{n!}(\partial_{y_i} \otimes \partial_{x_i})^n]$

e.g. $x_i * y_i = x_i y_i$, $y_i * x_i = y_i x_i + 1$

Localising, $y_i^{-1} * x_i^{-1} = \sum_{n \geq 0} (n!) y_i^{-(n+1)} x_i^{-(n+1)}$, and

$\mathcal{E}_{T^*V}(U) := (\widehat{\mathbb{C}[U]}, *)$ where we complete to include $\sum_{j \leq i}^{\infty} a_j$, $a_j \in \mathbb{C}[U]_j$.

Then $D(V) \xrightarrow{\sim} (\mathbb{C}[T^*V], *)$, via $x_i \mapsto x_i$, $\partial_i \mapsto y_i$.

Theorem: Let X be a symplectic variety with a compatible $\mathbb{C}^\times$-action, scaling ω_X positively. Then:

(i) $\exists$ $\operatorname{Per}: \underset{\sim}{\operatorname{Quant}(X)} \longrightarrow H^2_{dR}(X, \mathbb{C})$

(not $\mathbb{C}^\times$-equivariant: $\underset{\sim}{\operatorname{Quant}(X)} \longrightarrow [\omega_X] + h H^2_{dR}(X, \mathbb{C})[[h]]$ as $\mathbb{C}[[h]]$-algebras

(ii) if $H^i(X, \mathcal{O}_X) = 0$ for $i > 0$ then Per is a bijection.

Note that in case $f: X \to Y$ is a symplectic resolution, then $H^i(X, \mathcal{O}_X) = 0$ for $i > 0$ by Grauert-Riemenschneider. Furthermore, multiplication by -1 in $H^2_{dR}(X, \mathbb{C})$ corresponds to the sheaf morphism $\mathcal{E} \to \mathcal{E}^{op}$.

Suppose G has a hamiltonian action on Z, which commutes with the $\mathbb{C}^\times$-action on Z. Let $\mathcal{U}$ quantise Z G-equivariantly (e.g. $T^*R \to D(R)$). Suppose the moment map $\mu^\#: \mathfrak{g} \to \mathbb{C}[Z]$ satisfies $\{\mu^\#(A), -\} = \xi_A$, a differential action on $\mathbb{C}[Z]$.
We also need $\Phi: \mathfrak{g} \to \mathcal{U}$ s.t. $[\Phi(A), -] = \xi_A$ (derivation of $\mathcal{U}$).

<u>Definition:</u> Let Φ be as above and $\lambda \in (\mathfrak{g}^*)^G$. The <u>quantum hamilton reduction</u> is: $\mathcal{U} /\!/\!/_\lambda G := \left(\dfrac{\mathcal{U}}{(\Phi(A)+\lambda(A) \mid A \in \mathfrak{g})\mathcal{U}} \right)^G$.

Since $\mathcal{U}$ quantises Z, we can rewrite this as:

$$\left(\frac{\mathbb{C}[Z]}{(\mu^{\#}(A) \mid A \in \mathfrak{g})\,\mathbb{C}[Z]} \right)^G = \mathbb{C}\left[\mu^{-1}(0)/G\right]$$

$$\left(\frac{\operatorname{gr} G}{\operatorname{gr}(\Phi(A)+\lambda(A))\operatorname{gr}\mathcal{U}} \right)^G \twoheadrightarrow \operatorname{gr}\left(\mathcal{U} /\!/\!/_\lambda G\right).$$

This map is an isomorphism in general circumstances (flatness of μ)

<u>Example:</u> $G(\underline{v})$ acts on $T^*\operatorname{Rep}(Q, \underline{v}, \underline{w}) = \{(x_a, x_{a^*}, i_k, j_k)\}$
$\in \operatorname{Mat}(v_{t(a)}, v_{h(a)})$.

Let $\mathfrak{gl}(v_k) = \operatorname{Lie}(G(\underline{v}))$, $E^{(k)}_{uv} \in \mathfrak{gl}(v_k)$ then:

$$\Phi : E^{(k)}_{uv} \longmapsto \frac{1}{2}\left[\sum_{\substack{a \in Q_1 \\ t(a)=k}} \sum_{r=1}^{v_{h(a)}} \left(x_{a,rv}\,\partial_{a,ru} + \partial_{a,ru}\,x_{a,rv}\right) - \sum_{\substack{a \in Q_1 \\ h(a)=k}} \sum_{s=1}^{v_{t(a)}} \left(x_{a,us}\,\partial_{a,vs} + \partial_{a,vs} + x_{a,us}\right) + \sum_{r=1} \left(i_{k,rv}\,\partial_{k,ru} + \partial_{k,ru}\,i_{k,rv}\right) \right]$$

Then $\mathcal{E}^{\theta}_{\lambda}(Q, \underline{v}, \underline{w}) := \mathcal{E}_{T^*R}\big|_{T^*R^{\theta\text{-}ss}} /\!/\!/_\lambda G$ is a quantisation of $\mathcal{M}^{\theta}_{\lambda}(Q, \underline{v}, \underline{w})$

The parameter $\lambda \in (\mathfrak{g}^*)^G$ gives line bundles on $\mathcal{M}^{\theta}(Q, \underline{v}, \underline{w})$, and hence a class in $H^2_{dR}(\mathcal{M}^{\theta}(Q, \underline{v}, \underline{w}), \mathbb{C})$. This is an isomorphism for finite and affine quivers.

Exercises: (i) Recall the quiver $1 \to 2 \to \dots \to n-1$. $\mathcal{E}_\lambda^{(-1\dots-1)}(Q, \underline{v}, \underline{w})$ quantises T^*Fl with $\lambda \in (\mathfrak{g}^*)^G = \mathbb{C}^{Q_0} = \mathbb{C}^{n-1}$: $\mathbb{C}^{n-1} \to H^2(T^*Fl, \mathbb{C}) = \mathfrak{h}^*$.
There are twisted differential operators on Fl labelled by $\mathfrak{h}^*$:

$$\mathcal{E}_\lambda^{(-1,\dots,-1)} \longleftrightarrow D_{FL}^{-(\Sigma\lambda_i w_i)-\rho}.$$

(ii) Recall the quiver $\circlearrowleft$ $\mathcal{E}_\lambda^{-1}(Q, \underline{v}, \underline{w})$ quantises the Hilbert scheme $Hilb^n(\mathbb{C}^2)$, and global sections of $\mathcal{E}_\lambda^{-1}$ are rational Cherednik algebras (spherical part) of type A_{n-1}.

Lecture 3

Let Z be an affine scheme and $\mathcal{U}$ a filtered quantisation of $\mathbb{C}[Z]$. Given $M \in \mathcal{U}$-mod, filter it by $(\dots \subseteq F^jM \subseteq F^{j+1}M \subseteq \dots)$ s.t. $(F^k\mathcal{U})\cdot(F^jM) \subseteq F^{k+j}M$. Then $\mathrm{gr}\, M$ is a $\mathrm{gr}\, \mathcal{U}$-module.

Definition A good filtration on M is a (complete and separated) filtration s.t. $\mathrm{gr}\, M$ is a finitely generated $\mathbb{C}[Z]$-module.
The characteristic variety of M is $V(M) := \mathrm{supp}\, \mathrm{gr}\, M$, a closed subvariety of Z

Definition: Suppose X is a symplectic variety with a $\mathbb{C}^\times$-action. A sheaf of $\mathcal{E}_X$-modules M is coherent if $\mathrm{gr}\, M$ is a coherent $\mathcal{O}_X$-module. The category of coherent $\mathcal{E}_X$-modules is denoted $\mathcal{E}_X$-mod.

Let $f: X \to Y$ be a conical symplectic resolution. There is the global section functor $\Gamma: \mathcal{E}_{X,\lambda}\text{-mod} \longrightarrow (\mathcal{E}_{X,\lambda}(X) = \mathcal{U}_\lambda)\text{-mod}: M \longmapsto \Gamma(M)$, for $\lambda \in H^2(X, \mathbb{C})$.
This has a left adjoint $\mathrm{Loc} := \mathcal{E}_{X,\lambda} \otimes_{\mathcal{U}_\lambda} -$, the localisation functor.
Derived localisation holds for $\lambda \in H^2(X, \mathbb{C})$ if $R\Gamma$ is an equivalence.
Theorem: $\mathcal{U}_\lambda$ has finite global dimension iff derived localisation holds.
Localisation holds for $\lambda \in H^2(X, \mathbb{C})$ if Γ is an equivalence.

$\mathcal{E}_{X,\lambda}$ admits another description if $X = T^*V /\!\!/\!\!/_\theta G = \mu^{-1}(0)^{\theta-ss}/G$. Let $(D(V), G, \lambda)$-mod denote the category of G-equivariant $D(V)^G$-modules M s.t. $\xi_M = \Phi + \lambda$, where ξ_M is the differential of the G-action. We have a functor:

$$\begin{array}{ccc} (D(V), G, \lambda)\text{-mod} & \xrightarrow{\mathbb{H}_\lambda} & \left(\dfrac{D(V)}{(\Phi+\lambda)D(V)}\right)^G\text{-mod} \\ \Psi & & \Psi \qquad = \mathcal{U}_\lambda \\ M & \longmapsto & M^G \end{array}$$

$\mathbb{H}_\lambda$ is the <u>Hamiltonian reduction functor</u>. This yields an equivalence of categories: $\mathcal{E}_{X\lambda}\text{-mod} \simeq \dfrac{(D(V), G, \lambda)\text{-mod}}{(D(V), G, \lambda)\text{-mod}^{\theta\text{-uns}}}$.

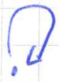
where $(D(V), G, \lambda)\text{-mod}^{\theta\text{-uns}}$ is the full subcategory of $(D(V), G, \lambda)$-modules whose characteristic variety belongs to $(T^*V)^{\theta\text{-uns}} := T^*V \setminus (T^*V)^{\theta\text{-ss}}$.

Hence we have a commutative diagram:

$$\begin{tikzcd} \dfrac{(D(V), G, \lambda)\text{-mod}}{(D(V), G, \lambda)\text{-mod}^{\theta\text{-uns}}} \arrow[r, "\Gamma"] \arrow[d] & \mathcal{U}_\lambda\text{-mod} \\ (D(V), G, \lambda)\text{-mod} \arrow[ur, "\mathbb{H}_\lambda"'] & \end{tikzcd}$$

<u>Theorem</u>: Localisation holds generically, and for some specific cases based on Kirwan–Ness stratification.

Now let $f: X \to Y$ be conical, with a $\mathbb{C}^\times$-action to create category $\mathcal{O}$.

<u>Example</u>: Consider the quiver . Then $\mathcal{M}^1(Q, n, 1) =$

$$\frac{\{(x, y, i, j) \in \mathrm{Mat}_n(\mathbb{C})^2 \times (\mathbb{C}^n)^* \times \mathbb{C}^n}{GL_n(\mathbb{C})}$$

$$[x, y] + ji = 0,\ \nexists\, 0 \neq \frac{\mathbb{C}^n}{S} \text{ with } (x, y)(S) \subseteq S \text{ and } \operatorname{im} j \subseteq S \text{ (Stability)}\}$$

$\simeq \mathrm{Hilb}^n(\mathbb{C}^2) = \{I \trianglelefteq \mathbb{C}[x, y] \mid \text{colength } I = n\}$.

(i) x, y commute on $j(1)$ since $xyj(1) = yxj(1) - \cancel{jij(1)}$ 0 by $\mathrm{trace}(ij) = 0$.

(ii) $I = \mathrm{Ann}_{\mathbb{C}[x,y]}\, j(1)$

(iii) stability condition force colength $I = n$.

Institute of Science and Technology Austria | Am Campus 1 | 3400 Klosterneuburg | Austria
Phone +43 (0) 2243/9000 | Fax +43 (0) 2243/9000 - 2000 | office@ist.ac.at | www.ist.ac.at

Let $f: X = \mathrm{Hilb}^n \mathbb{C}^2 \longrightarrow \mathrm{Sym}^n \mathbb{C}^2$ be defined by $f(I) = \mathrm{Supp}(I)$. There is a $\mathbb{C}^\times$-action on $\mathrm{Sym}^n \mathbb{C}^2$ given by $t \cdot \{(x_1, y_1), \ldots, (x_n, y_n)\} = \{(t x_1, t^{-1} y_1, \ldots, t x_n, t^{-1} y_n)\}$.

Let $X_+ := \{x \in X \mid \lim_{t \to 0} t \cdot x \text{ exists}\}$, $Y_+ = \{(x_1, 0), \ldots, (x_n, 0)\}$. Then $X_+ = f^{-1}(Y_+)$, and $X_+ = \bigcup_\lambda X_+^\lambda$ is a union of irreducible lagrangians. There is a partial ordering (by closure) on the components. Set $\mathrm{Sm}(X_+^{\min}) = N$.

<u>Category $\mathcal{O}_\lambda$</u> is the $\mathbb{C}^\times$-equivariant $\mathcal{E}_{X,\zeta}$ modules with support on X_+.

Here, $N = f^{-1}(\{(x_1, 0), (x_2, 0), \ldots, (x_n, 0) \mid x_i \neq x_j \text{ for } i \neq j\}) = f^{-1}\left(\frac{\mathbb{C}^n_{reg}}{S_n} \times \{0\}\right)$
$\simeq \frac{\mathbb{C}^n_{reg}}{S_n}$.

Restricting sheaves from X_+ to N yields local systems on N, and a functor $\mathbb{V}: \mathcal{O}_\lambda \longrightarrow \pi_1(N)\text{-mod}$ obtained by taking monodromy.

<u>Theorem</u>: In the case $\mathcal{R}$, the functor $\mathbb{V}: \mathcal{O}_\lambda \longrightarrow \pi_1(N) = Br_n\text{-mod}$ factors through the Hecke algebra $\mathcal{H}_q(S_n)$-mod. Let $q = e^{2\pi i \lambda}$; then $\mathcal{O}_\lambda$ is equivalent to representations of the q-Schur algebra.

$\mathbb{V}$ is the <u>KZ-functor</u>, isomorphic to the Schur functor.

<u>Exercise</u>: Understand $\mathbb{V}$ in other examples.