

EXERCISE SHEET 1

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1. Calculate $\text{Hilb}_n(\mathbb{P}^1)$.
2. Compute $\text{Tan}_{\mathcal{I}}(\text{Hilb}(S))$ for S a smooth surface and \mathcal{I} a finite-colenlength ideal. Prove that if S is a $K3$ surface, then $\text{Hilb}(S)$ is a holomorphic symplectic manifold.
3. Suppose that operators $A^*(\text{Hilb}) \xrightarrow{a_n} A^*(\text{Hilb} \times S)$ satisfy the following (Heisenberg) relation:

$$[a_n, a_{n'}] = n\delta_{n+n'}^0 \Delta_* \Pi^*.$$

Compute

$$[a_n a_m(\Delta), a_{n'} a_{m'}(\Delta)].$$

What is the algebra generated by the operators

$$a_n a_m(\Delta): A^*(\text{Hilb}) \rightarrow A^*(\text{Hilb})?$$