EXERCISE SHEET 1

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1. Compute the following for the type A nilpotent x of Jordan type $(n-1,1)$.
   (a) Closures of the attracting cells for the $C_T(x)$ action on the Springer fibre $\mathscr{B}_x$.
   (b) The convolution algebra A^1_{conv}.
   (c) Compute the centre of A^1_{conv}.
   (d) Show the following equivalence:
   $$\mathscr{O}_0^{\mathfrak{p}_x}(\mathfrak{sl}_n) \cong A^1_{\mathrm{conv}} - \mathrm{mod}.$$
2. Compute the irreducible components of the Springer fibre $\mathscr{B}_x$ when x is a type $(2,2)$ nilpotent.
3. Show that $T^*(G/B) \cong \{(\mathfrak{b}, x) \mid x \in \mathfrak{n}\}$.
4. For the type $(1,1)$ nilpotent, show that
   $$A^1_{\mathrm{conv}} \cong \mathrm{Perv}(\mathbb{P}^1).$$
5. Find out the dimension of $H^*(\mathscr{B}_x)$ in type A.
6. Let $\mathscr{C}$ be a category. Then $\mathrm{Aut}(\mathscr{C})$ is the group of all functors $F\colon \mathscr{C} \to \mathscr{C}$ that are autoequivalences, up to isomorphism. Compute this group in the following cases.
   (a) The category of sets.
   (b) The category of abelian groups.
7. Let $\mathscr{A}$ be an abelian category.
   (a) Show that
   $$K_0(\mathscr{A}) \cong K_0(D^b(\mathscr{A})).$$
   (b) What is $K_0(D^-(\mathscr{A}))$?