# EXERCISE SHEET 2

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1. Recall that if $\mathscr{A}$ is an abelian category, then $Z(\mathscr{A}) = \mathrm{End}(\mathrm{id})$. Show that $Z(A\text{-mod}) = Z(A)$.
2. Show that the braid group action on $\mathscr{O}_0(\mathfrak{sl}_2)$ by shuffling functors is faithful.
   (Hint: Let $T_s = \mathrm{Cone}(\mathrm{id} \xrightarrow{\mathrm{adj}} \Theta_s)$. Apply this to good modules, and then show that it categorifies the regular module for $\mathbb{C}[S_2]$.)
3. Show for $A = A^1_{\mathrm{conv}}$ in the case of the $(n-1,1)$ nilpotent, that
$$Ae_i \otimes e_i A \xrightarrow{\mathrm{mult}} A$$
   is a tilting complex in $D^b(A)$.
   (a) Compute its action on modules.
   (b) Define an inverse of the induced derived equivalence.
   (c) Put a grading on $A$ by setting $H^*(C_i) = H^*(\mathbb{P}^1)$ to be in degrees 0 and 2, and $H^*(C_i \cap C_j)$ to be in degree 1 if it is one-dimensional. With this grading, compute the induced action on $\mathscr{K}_0(A\text{-grmod})$.
4. Compute the indecomposable tilting modules $T(\lambda)$ for $\mathscr{O}_0(\mathfrak{sl}_2)$, and show that
$$T = \bigoplus_{\lambda} T(\lambda)$$
   is a tilting module in the sense of Happel. Show that $Z(\mathscr{O}_0(\mathfrak{sl}_2) = Z(\mathrm{End}_g(T))$, and compute it explicitly.
5. Show that the braid group action factors through a Weyl group action in the cases described.
6. Given a highest weight category of finite global dimension, give a construction to produce modules which have $\Delta$ and $\nabla$ flags, and show that they are tilting.